Local Terminations and Distributed Computability in Anonymous Networks

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Session SHAMAN

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So the question is

Given a *specification* S, a family of networks $\mathcal{F}$, is there a distributed algorithm $\mathcal{A}$ such that, any execution of $\mathcal{A}$ on any network in $\mathcal{F}$ ends with final state (labels) satisfying S.

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  - Reliability?

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Computability in resource-constrained autonomous large scale systems

- Evolution of the network,
- Impossibility results: properties of the network that have to be “centrally” maintained.
- Possibility results: with efficient algorithms?

Motivations for Local Termination

Different kinds of termination exist for distributed algorithms.

- Implicit termination: processes do not know that the computation is over.
  - [Boldi and Vigna '02] Characterization of tasks computable with implicit termination,
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  - equivalence with self-stabilizing (terminating) tasks.
- Local termination: at some point, each process knows that it has computed its final value.
- Global termination detection: processes know that all processes have computed their final value.
  - [C., Godard, Métivier, Tel ’07] Characterization of tasks computable with global termination detection
  - based on coverings and quasi-coverings

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- Weak Local Termination:
  - a process knows when it has computed its final value, but can still run the algorithm (processing and forwarding other nodes messages).

Applications

Two distributed algorithms are composed whenever the second uses as inputs the outputs of the first algorithm.

- **weak local termination:** composition of distributed algorithms,
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A General Framework

Termination appears a key parameter to unify distributed computability results in communication networks.

Older Related Works

- Angluin ('80)
- Yamashita & Kameda ('95)
- Boldi & Vigna ('99,'01)
- Mazurkiewicz ('97)
- Metivier et al (90's)
- ...

Model: Reliable Message Passing Systems with Transient Faults

A network is represented as a graph G with a port-numbering δ where each process can

- modify its state,
- send a message via port p,
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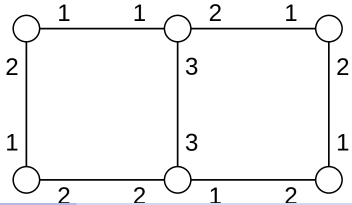

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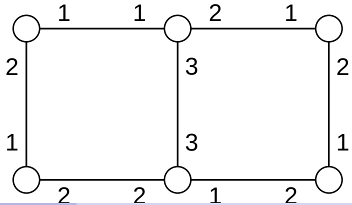

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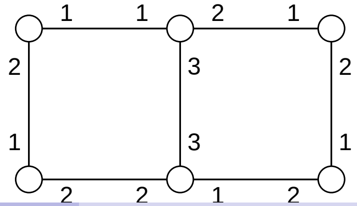

Examples of Tasks with Different Terminations

Problem

- $v \in V$ is labeled by $(\text{val}(v), \text{dist}(v)) \in \mathbb{N} \times \mathbb{N} \cup \{\infty\}$,
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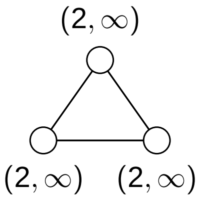

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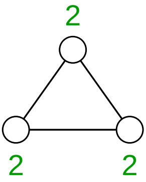

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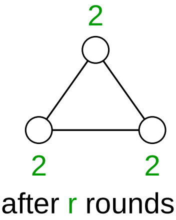

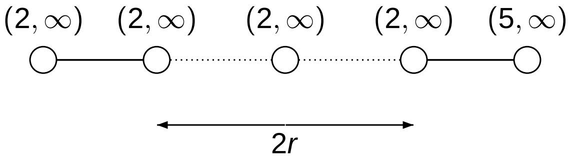

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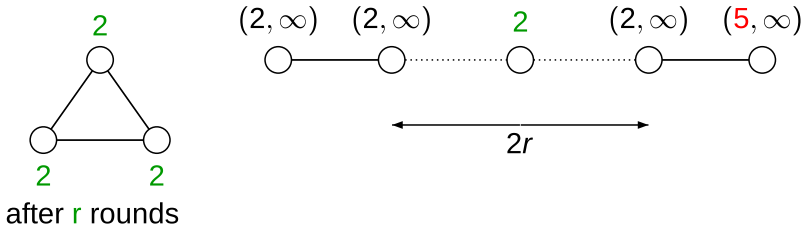

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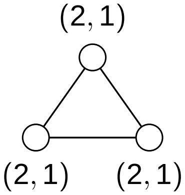

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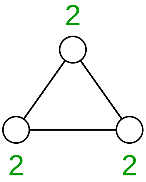

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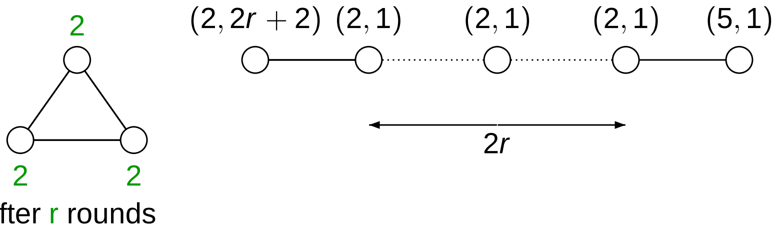

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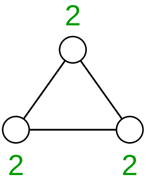

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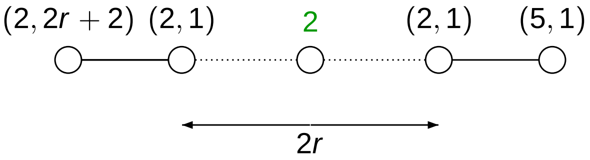

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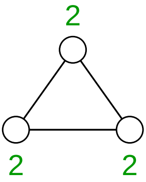

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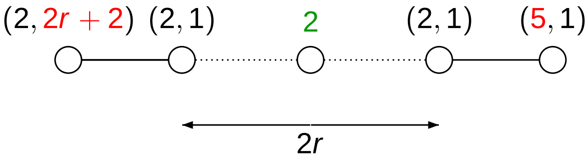

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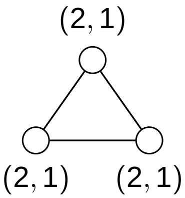

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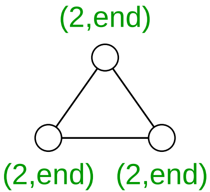

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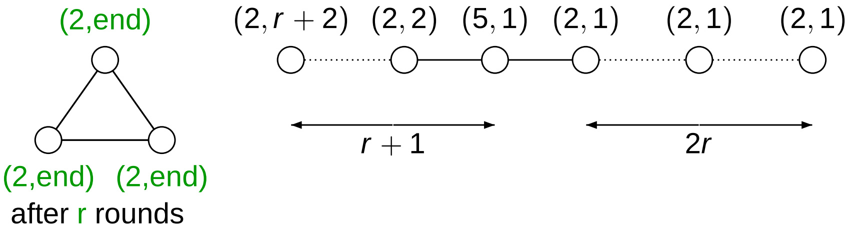

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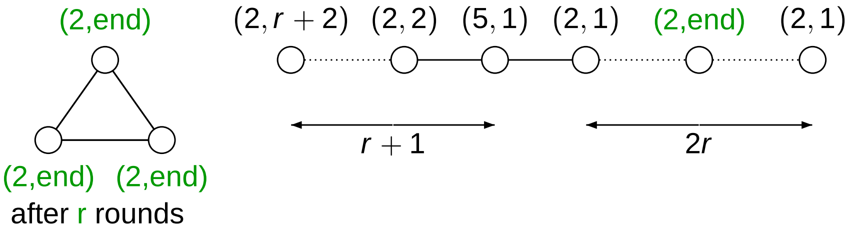

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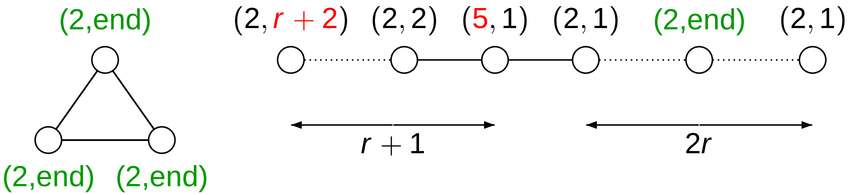

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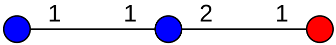
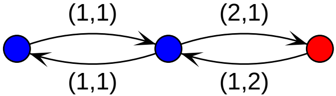
From graphs to digraphs

We represent the network by a labeled digraph (G, δ, λ) where the state of each process is encoded by its label.

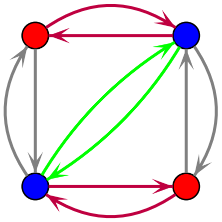




Coverings

Definition

D is a covering of D' via φ if φ is a locally bijective homomorphism.



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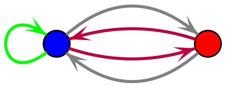

D'

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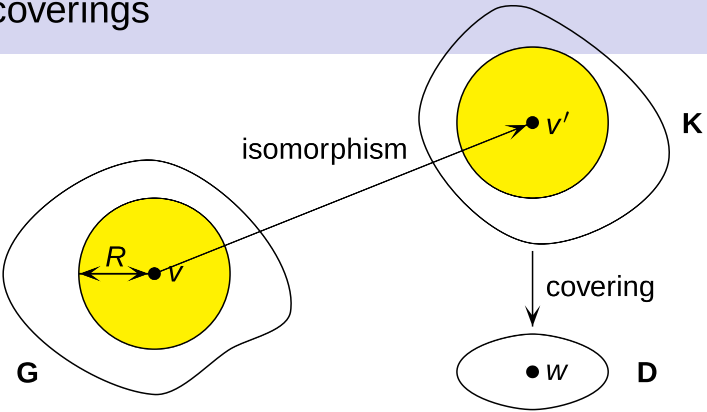
Lifting Lemma

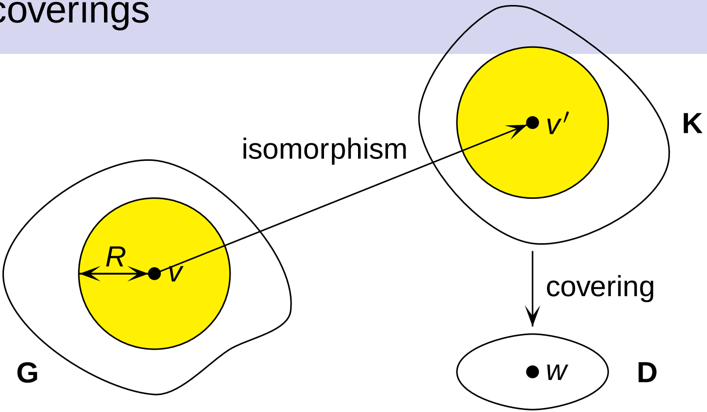
If **D** is a covering of **D**′ via φ, in the synchronous execution of any algorithm $\mathcal{A}$, v and $\varphi(v)$ behave in the same way.

Quasi-coverings



Definition: **G** is a quasi-covering of **D** of radius *R* of center *v*.

Quasi-coverings



Definition: **G** is a quasi-covering of **D** of radius R of center v.

Quasi-Lifting Lemma

In the synchronous execution of any algorithm $\mathcal{A}$, v and w behave in the same way during the first R rounds.

Necessary Condition

Suppose there exists an algorithm $\mathcal{A}$ that computes $(\mathcal{F}, S)$.

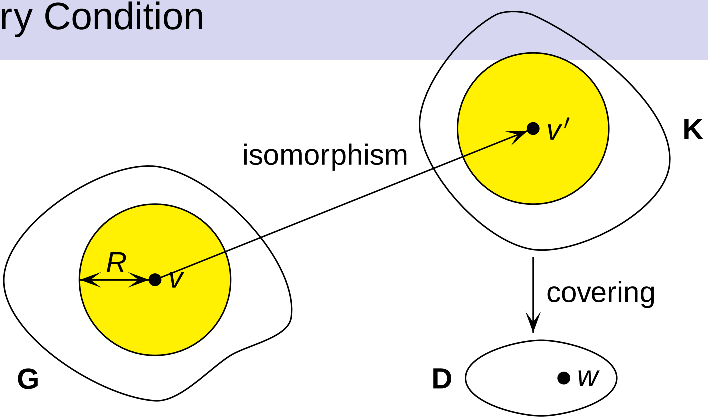
Given any digraph **G**, consider the synchronous execution of $\mathcal{A}$ over **G**.

For any $v \in V(G)$, let i be the first round (if it exists) where $\texttt{term}_i(v) = \textsc{Term}$, then we define:

- $\text{result}(\mathbf{G}, v) = \texttt{output}_i(v)$,
- $\text{time}(\mathbf{G}, v) = i$.

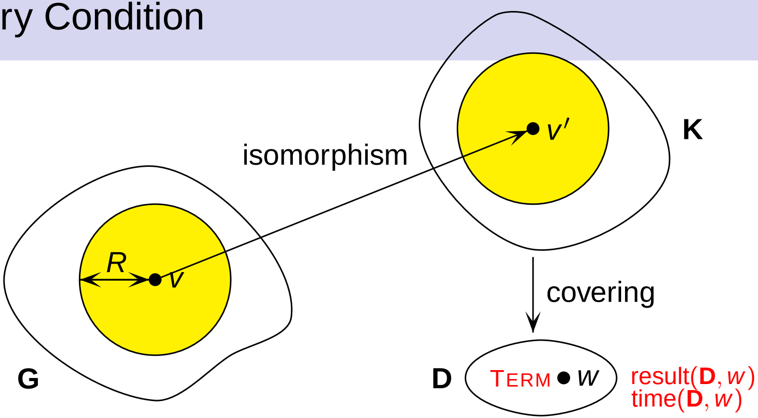
Otherwise, we let $\text{result}(\mathbf{G}, v) = \perp$ and $\text{time}(\mathbf{G}, v) = \infty$.

Necessary Condition



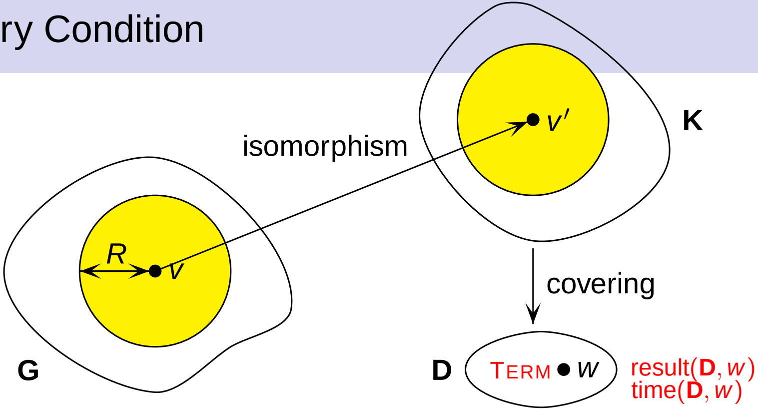
- During the first R rounds of the execution, v and w remain in the same state.

Necessary Condition



- During the first R rounds of the execution, v and w remain in the same state.
- If $R \geq \text{time}(\mathbf{D}, w)$ or $R \geq \text{time}(\mathbf{G}, w)$),
  $\text{result}(\mathbf{D}, w) = \text{result}(\mathbf{G}, v)$ and $\text{time}(\mathbf{D}, w) = \text{time}(\mathbf{G}, v)$.

Necessary Condition



Key Property

Two functions time and result have property (P) if for any $\mathbf{G}, \mathbf{D}$,

- if **G** is a quasi-covering of **D** of radius R
- if $R \geq \min\{\text{time}(\mathbf{G}, v), \textit{time}(\mathbf{D}, w)\}$

then $\text{time}(\mathbf{G}, v) = \text{time}(\mathbf{D}, v)$ and $\text{result}(\mathbf{G}, v) = \text{result}(\mathbf{D}, v)$

Weak Local Termination

Theorem

A task $(\mathcal{F}, S)$ can be computed on $\mathcal{F}$ with weak local termination

$$\Longleftrightarrow$$

there exists some functions time and result computable on $\mathcal{F}$ such that

- **G** *S result*(**G**)
- *time and result have Property (P)*

Weak Local Termination

Theorem

A task $(\mathcal{F}, S)$ *can be computed on* $\mathcal{F}$ *with weak local termination by a polynomial algorithm*

$$\Longleftrightarrow$$

there exists some functions time and result computable on $\mathcal{F}$ *such that*

- $\mathbf{G}\ S\ result(\mathbf{G})$
- *time and result have Property* (P)
- *there exists a polynomial p such that*
  - $\forall \mathbf{G} \in \mathcal{F}$, $\max\{time(\mathbf{G}, v) \mid v \in V(G)\} \leq p(|\mathbf{G}|)$

An Algorithm for the Sufficient Condition

A Mazurkiewicz-like Algorithm $\mathcal{M}$

When executed on a graph **G**,

- \+ it reconstructs **D** such that **G** is a covering of **D**
- \+ it is a polynomial algorithm

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+ at any step i, it enables each process v to reconstruct $\mathbf{D}_i(v)$ such that **G** is a quasi-covering of $\mathbf{D}_i(v)$ of center v

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+ at any step i, it enables each process v to reconstruct $\mathbf{D}_i(v)$ such that **G** is a quasi-covering of $\mathbf{D}_i(v)$ of center v

– v does not not know the radius of the quasi-covering

An Algorithm for the Sufficient Condition

Szymanski, Shy and Prywes Algorithm

Combined with our algorithm,

+ enables each v to compute a lower bound $\mathrm{lb}(v)$ on the radius of the quasi-covering
+ once $\mathcal{M}$ has implicitly terminated, this lower bound tends to ∞

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+ once $\mathcal{M}$ has implicitly terminated, this lower bound tends to ∞

– we have to be careful to avoid infinite computations

Computing the output

At each time step i, each v knows

- a graph $\mathbf{D}(v)$ such that $\mathbf{G}$ is a quasi-covering of $\mathbf{D}(v)$ of center v
- a vertex $w(v)$: the image of v in $\mathbf{D}(v)$
- a lower bound $\text{lb}(v)$ on the radius of the quasi-covering

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If $\text{lb}(v) \geq \text{time}(\mathbf{G}, v)$, then by Property (P)

- $\text{time}(\mathbf{G}, v) = \text{time}(\mathbf{D}(v), w(v))$
- $\text{result}(\mathbf{G}, v) = \text{result}(\mathbf{D}(v), w(v))$

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Problem

If $\mathrm{lb}(v) < \mathrm{time}(\mathbf{G}, v)$, $\mathrm{result}(\mathbf{D}(v), w(v))$ may be undefined if $\mathbf{D}(v)$ is not in $\mathcal{F}$

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Solution

- v enumerates the graphs of $\mathcal{F}$ until it finds a graph $\mathbf{H}$ that is a quasi-covering of $\mathbf{D}(v)$ of radius $\mathrm{lb}(v)$ of center u via γ such that $\gamma(u) = w(v)$

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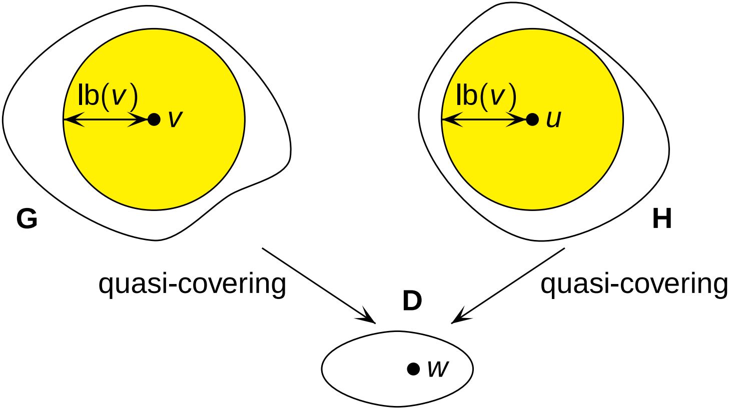
- If $\mathrm{time}(\mathbf{H}, u) \leq \mathrm{lb}(v)$,
  - $\texttt{output}(v) := \mathrm{result}(\mathbf{H}, u)$

Computing the output



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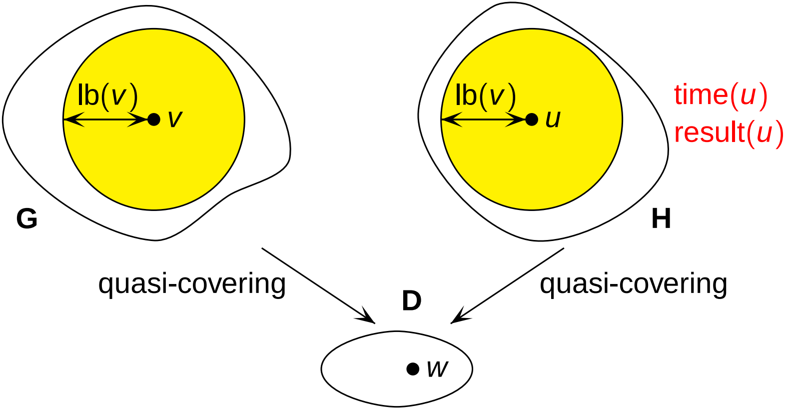

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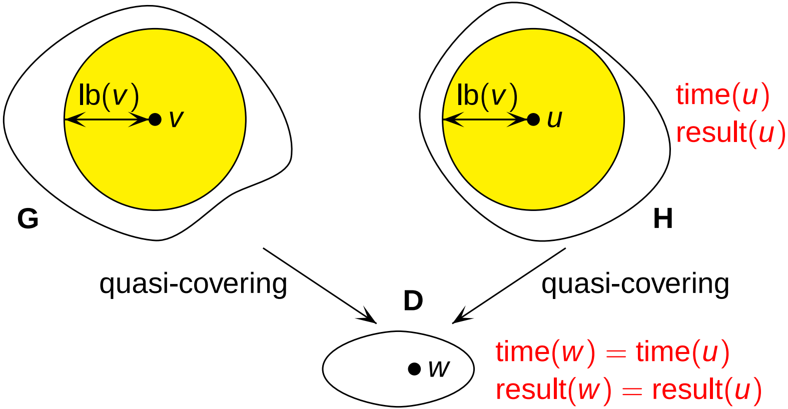

Since time(u) $\leq$ $lb(v)$, by Property (P)

- time(w) = time(u) and result(w) = result(v)

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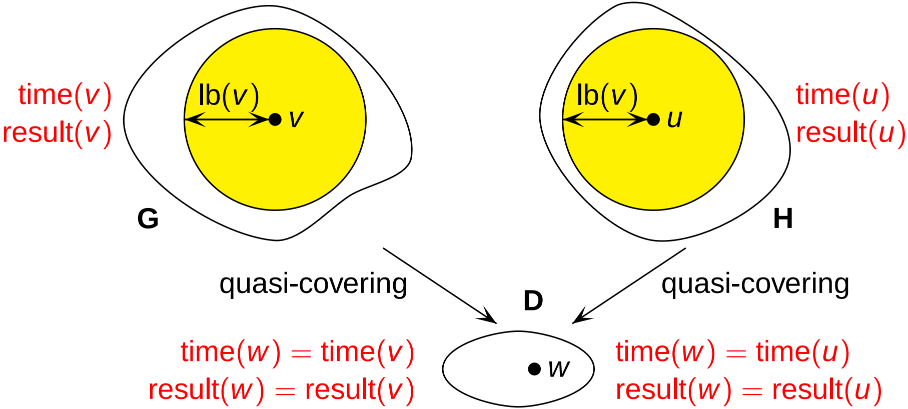

Since $\text{time}(u) \leq lb(v)$, by Property (P)

- $\text{time}(w) = \text{time}(u)$ and $\text{result}(w) = \text{result}(v)$
- $\text{time}(v) = \text{time}(u)$ and $\text{result}(v) = \text{result}(u)$

Complexity

Let $T(\mathbf{G}) = \max\{\text{time}(\mathbf{G}, v) \mid v \in V(G)\}$.

Proposition

Complexity of our Algorithm:

- $O(Dn + T(\mathbf{G}))$ *rounds*
- $O(m^2n + mT(\mathbf{G}))$ *messages*
- $O(\Delta \log n + \log T(\mathbf{G}))$ *bits per message*

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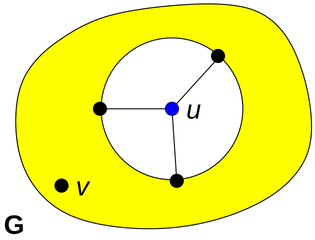
Corollary

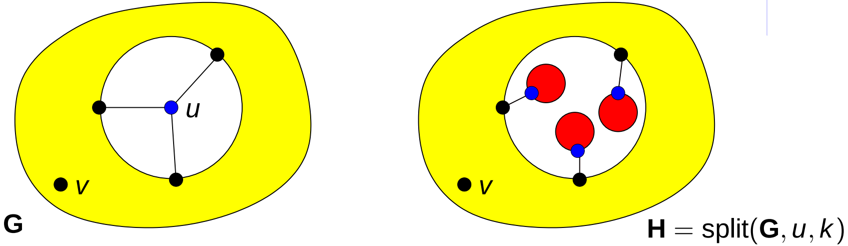
If there exists an algorithm $\mathcal{A}$ that computes a task $(S, \mathcal{F})$ in a polynomial number of rounds (even with big messages), there exists a polynomial algorithm $\mathcal{A}'$ that computes $(S, \mathcal{F})$

Necessary Condition for Local Termination



We define an operator split such that

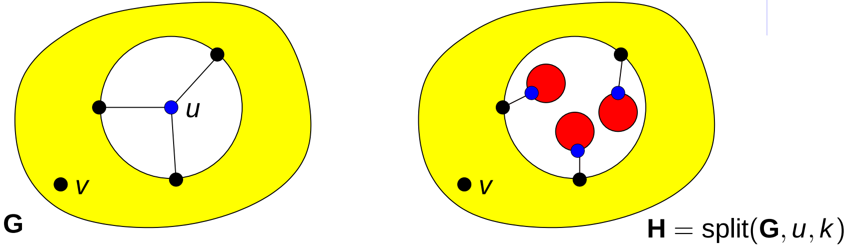
Necessary Condition for Local Termination



We define an operator split such that

- the blue vertices in split($\mathbf{G}, u, k$) behave like u in **G** during k steps

Necessary Condition for Local Termination



Property

Two functions time and result have property (P') if for any $\mathbf{G}$

- if $k = \text{time}(\mathbf{G}, u)$
- then, for any $v \neq u$, $\text{result}(\mathbf{G}, v) = \text{result}(\mathbf{H}, v)$ and $\text{time}(\mathbf{G}, v) = \text{time}(\mathbf{H}, v)$

Local Termination

Theorem

A task $(\mathcal{F}, S)$ can be computed on $\mathcal{F}$ with local termination

$$\iff$$

there exists some functions time and result computable on $\mathcal{F}$ such that

- $\mathbf{G}$ *S result*$(\mathbf{G})$
- *time and result have Property* (P) *and* (P')

Local Termination

Theorem

A task $(\mathcal{F}, S)$ can be computed on $\mathcal{F}$ with local termination by a polynomial algorithm

$$\Longleftrightarrow$$

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Thank you