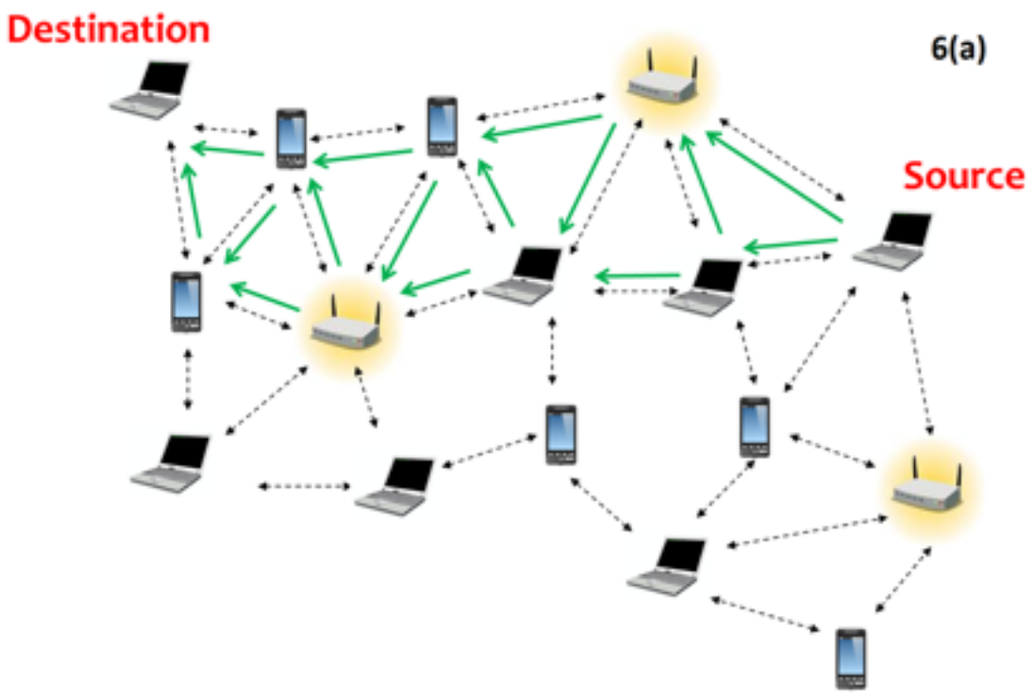
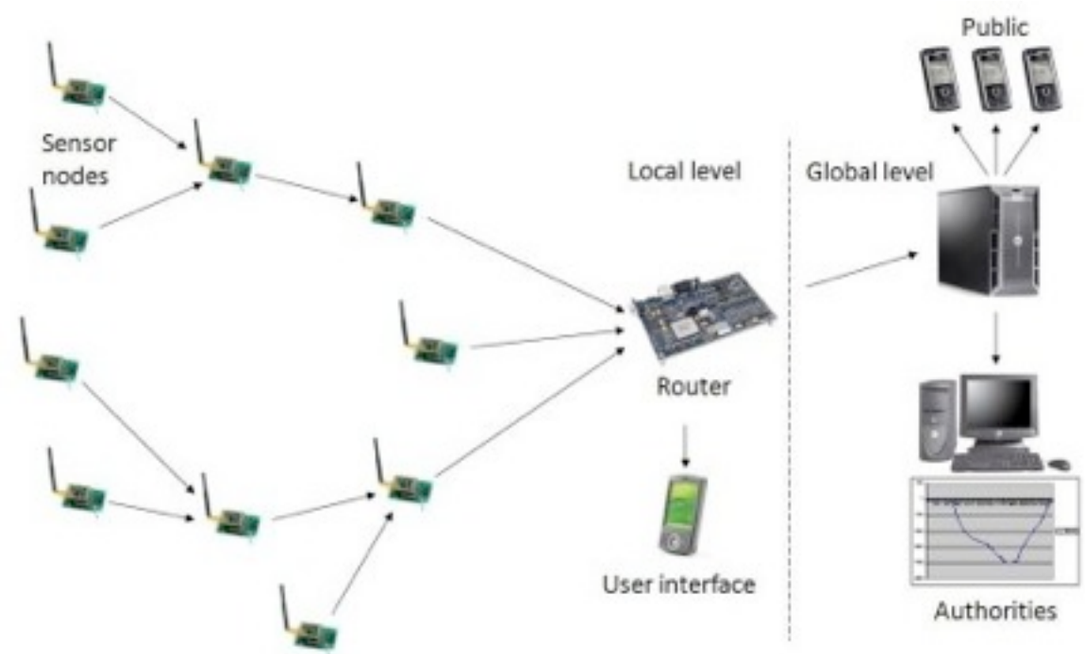
Trees & Routing (II)
MST

Why (minimum spanning) trees ?

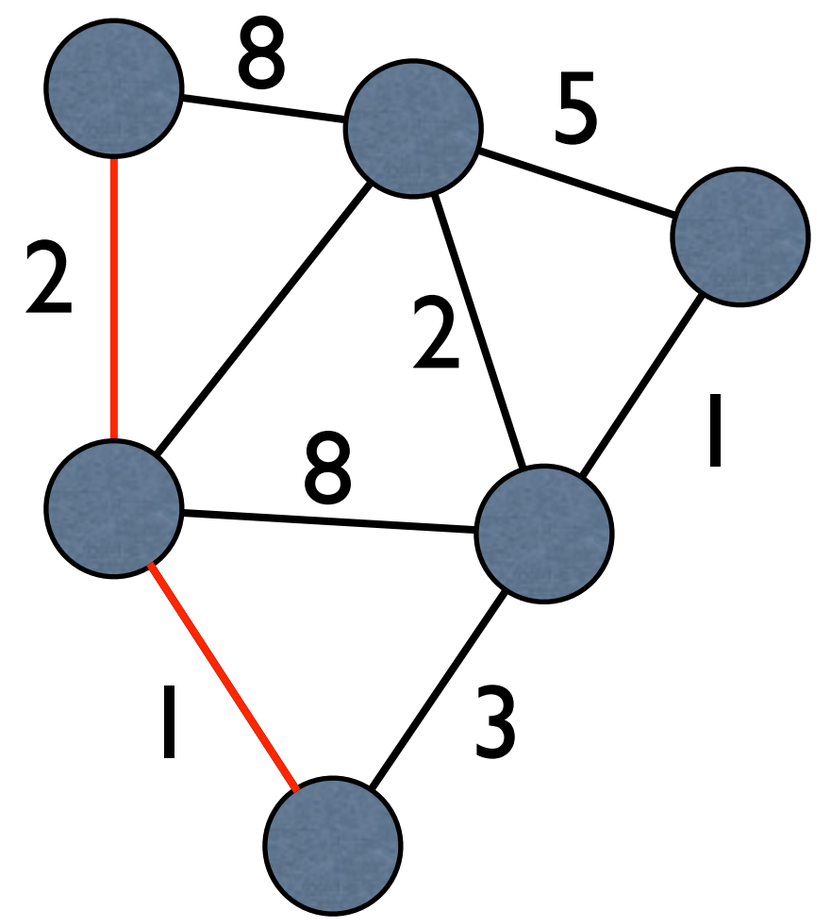




Minimum Spanning Tree

- Given a weighted graph G=(V,E,ω), a **MST** of G is a spanning tree of G of minimum weight.

- If weights in G are distinct then MST is unique ! (PROVE IT !)



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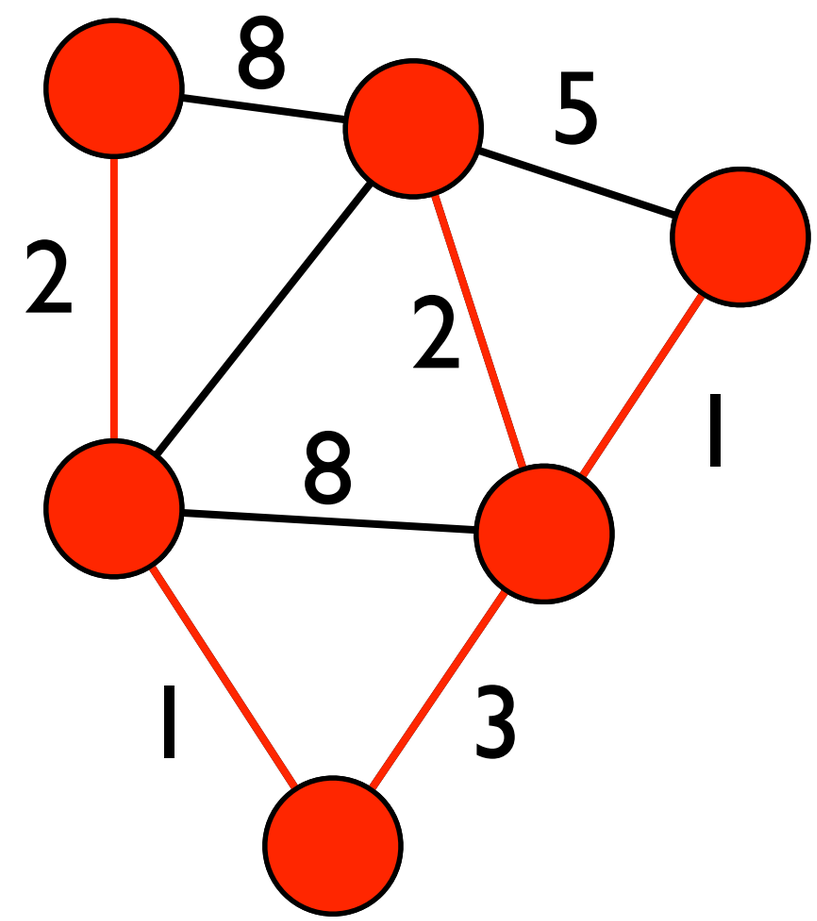
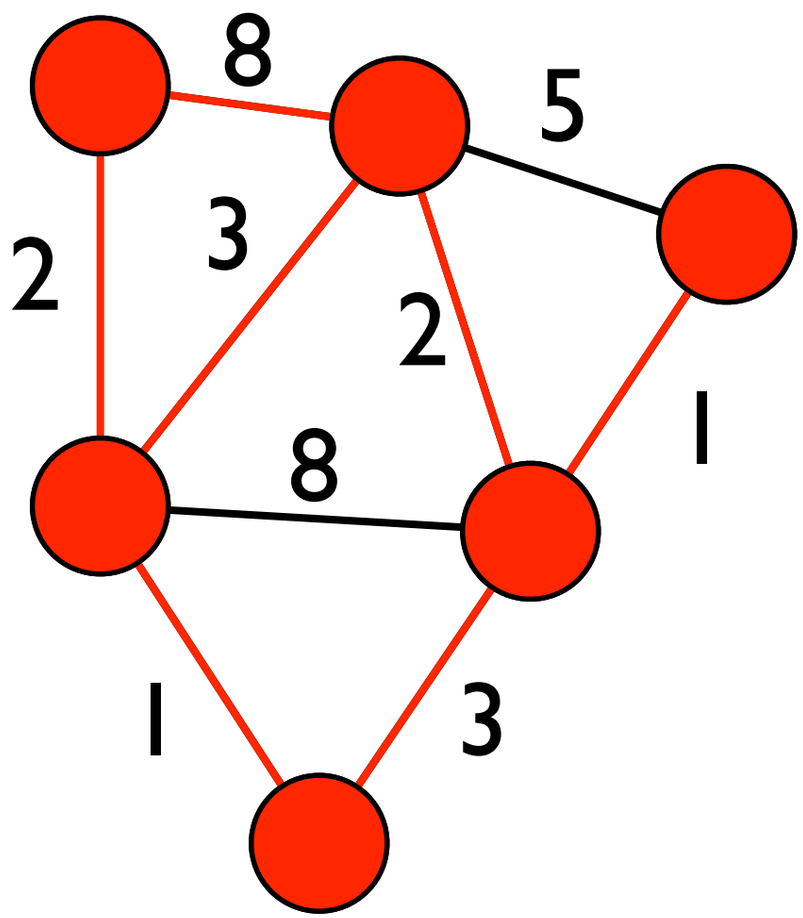
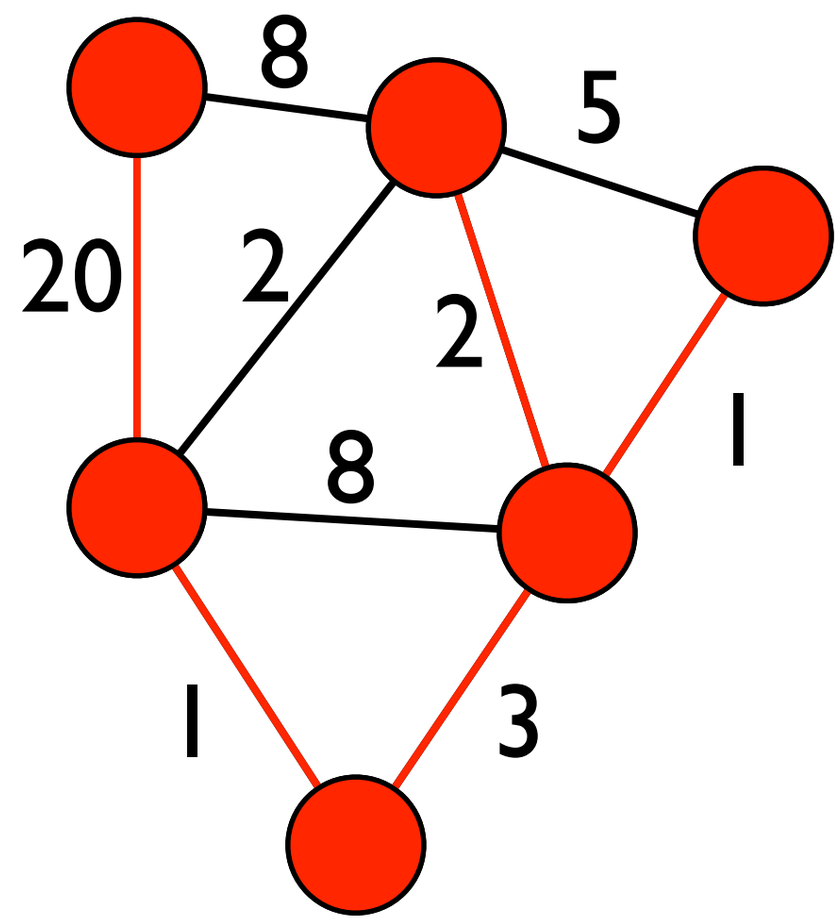


BFS or MST ?





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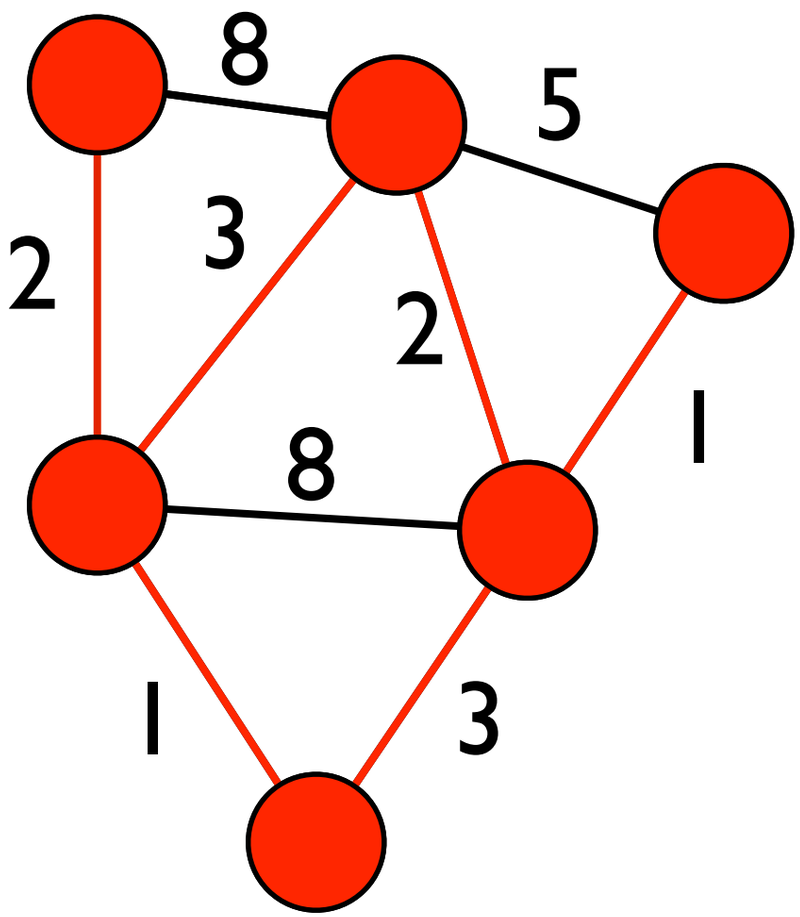
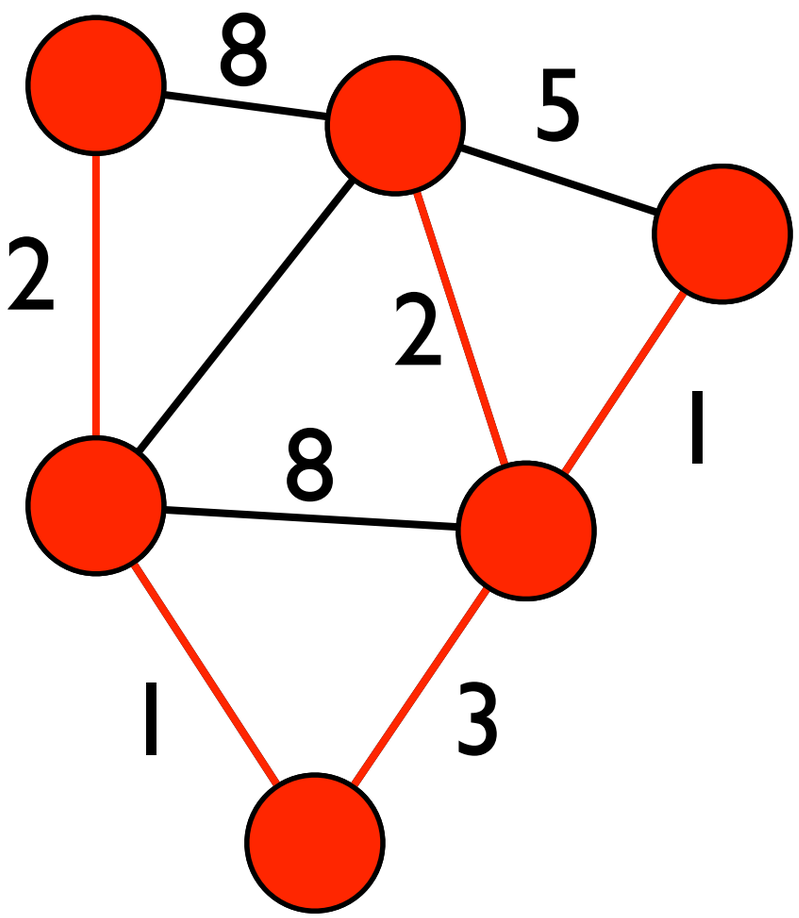




MST definitions

- Given a weighted graph G=(V,E,ω), a T in G is called MST *fragment* of G if exists an MST of G having T as subtree.

  Theorem : Given a weighted graph G=(V,E,ω) and a fragment T of G, the union between T and the *minimum weight outgoing edge* of T (*mwoe(T)*) is also a fragment.

MST Construction

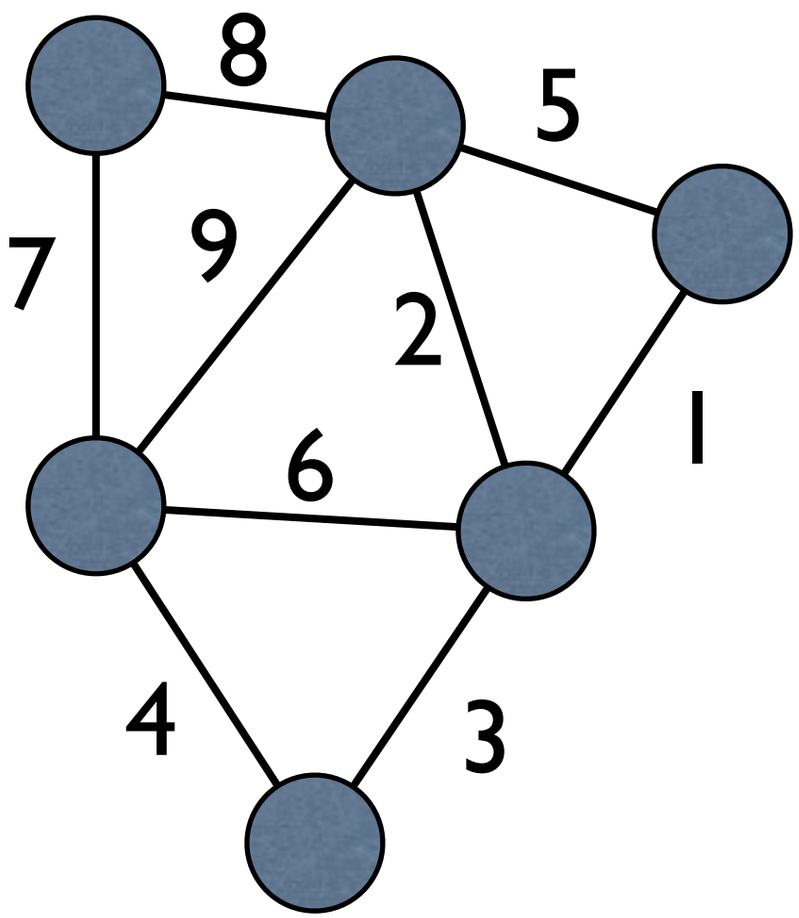
MST Strategies

- **blue rule :** Pick a fragment, T, and add to it the mwoe(T).
- **red rule :** Pick a cycle in the remaining graph and erase the heaviest edge in that cycle.

Blue Rule Strategies

Prim Algorithm

- the unique root is the initial fragment
- apply the «blue» rule until reaching the MST



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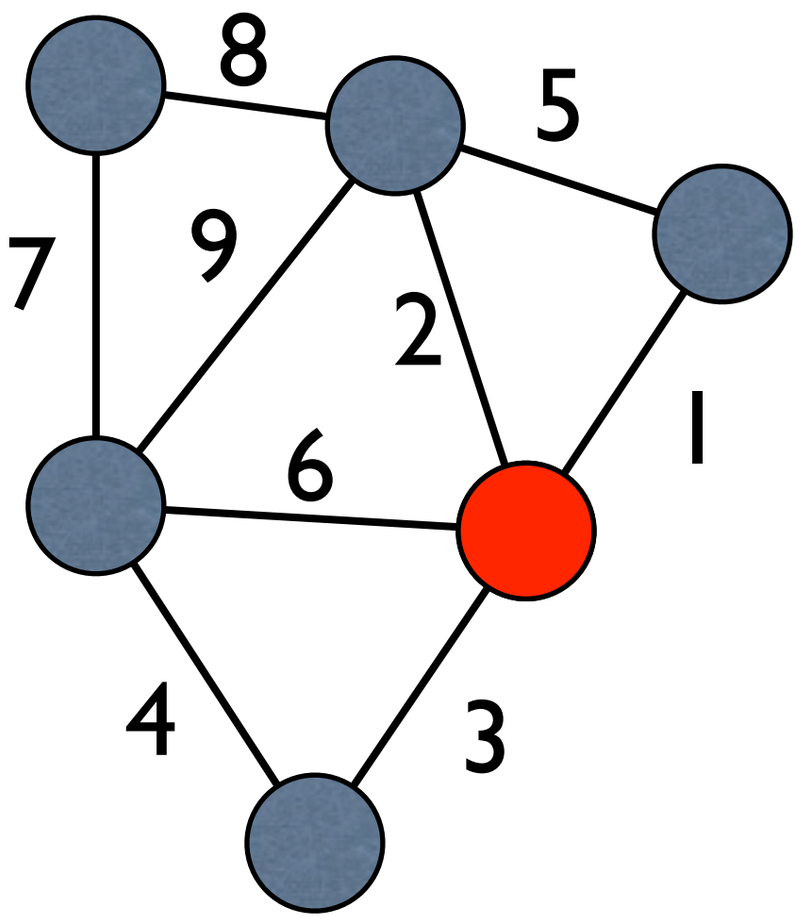

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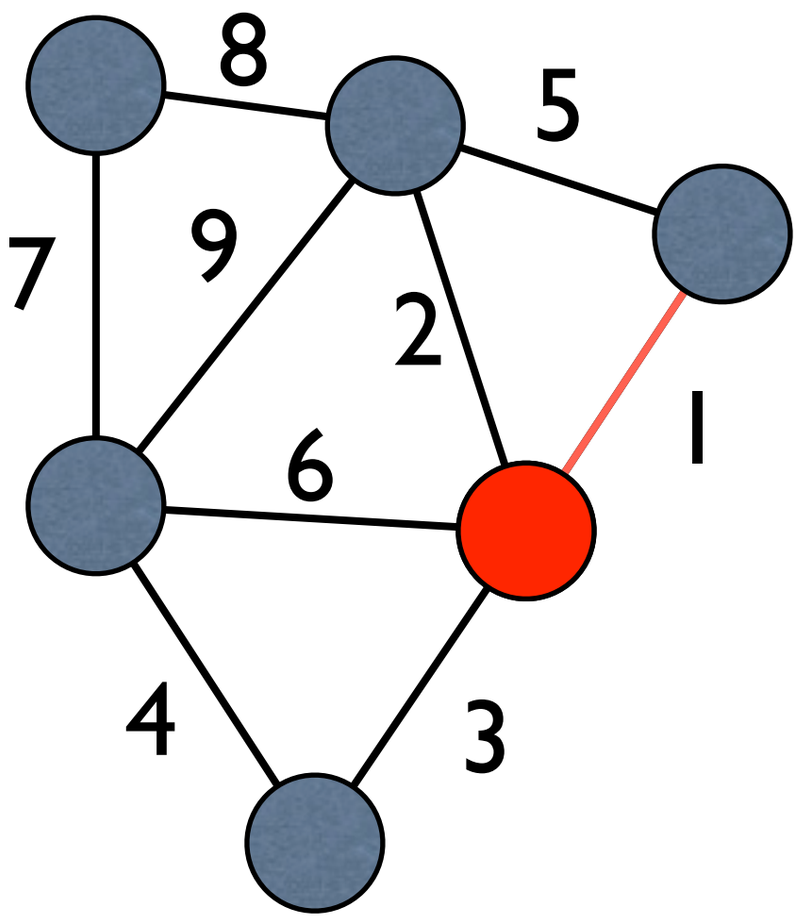

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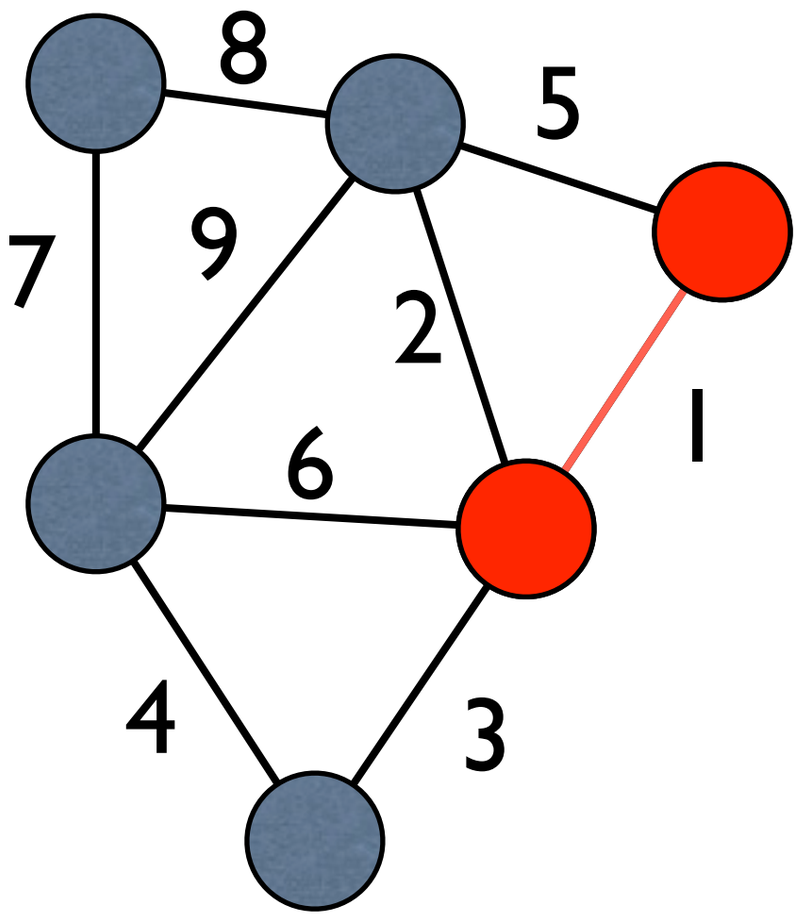

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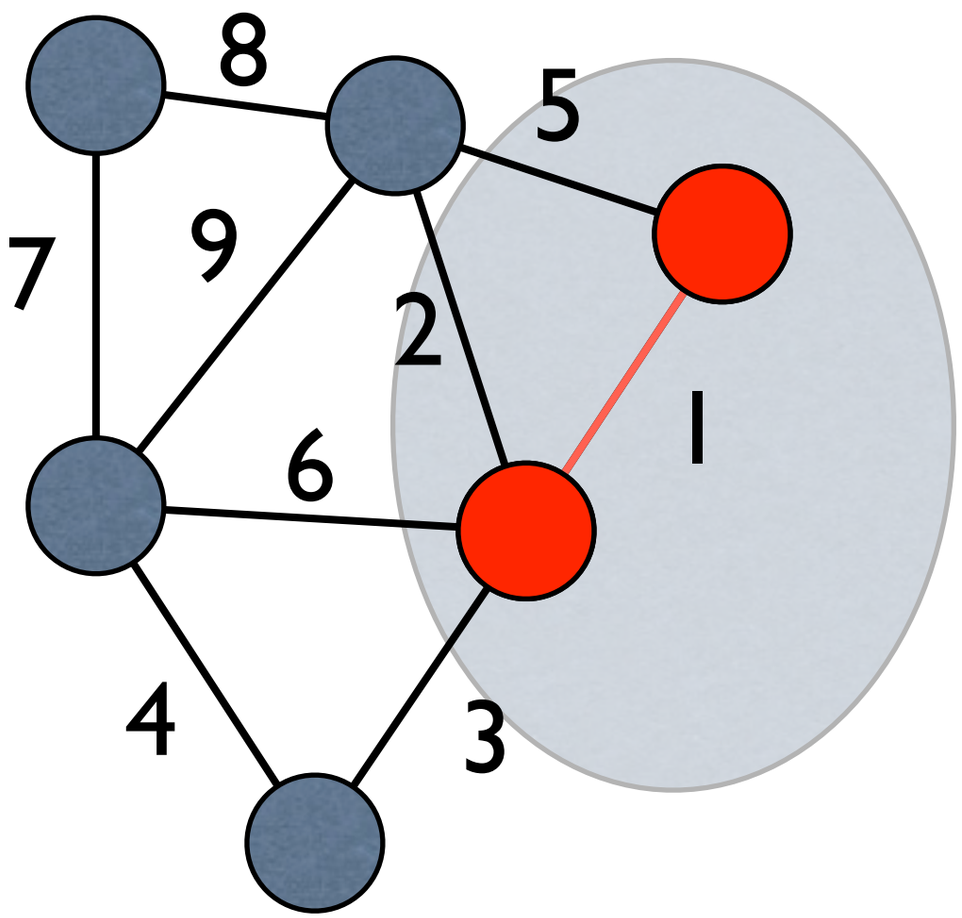

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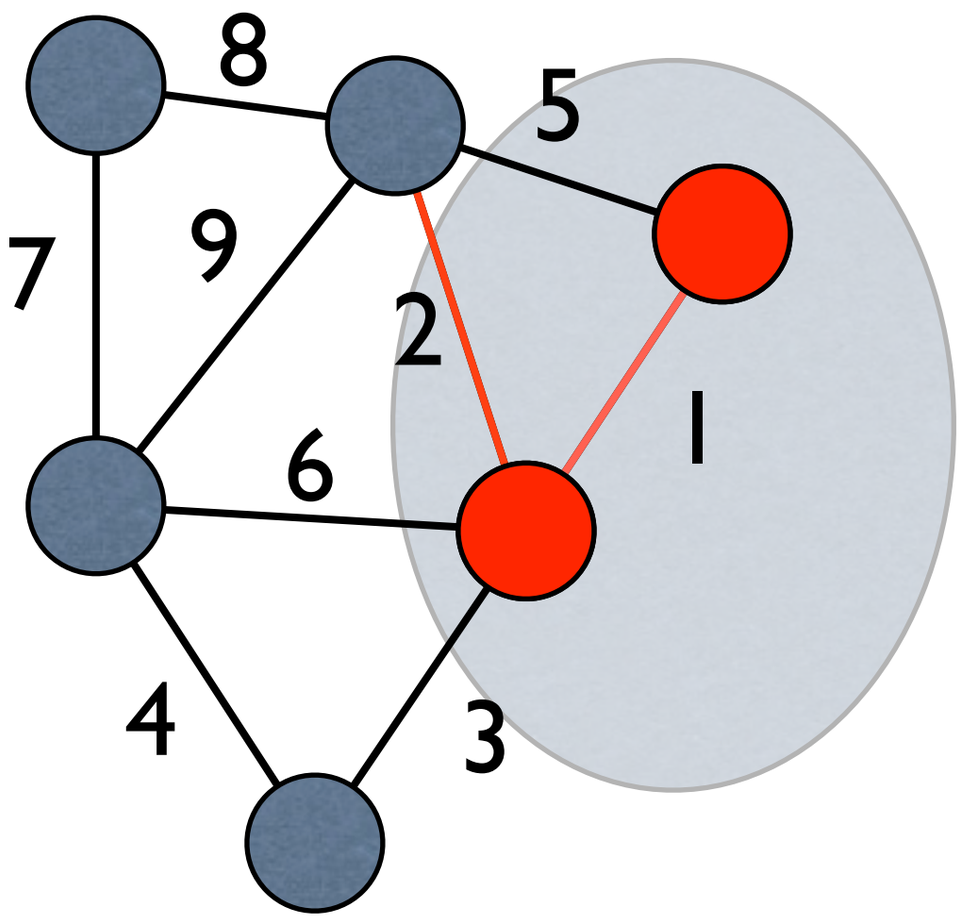

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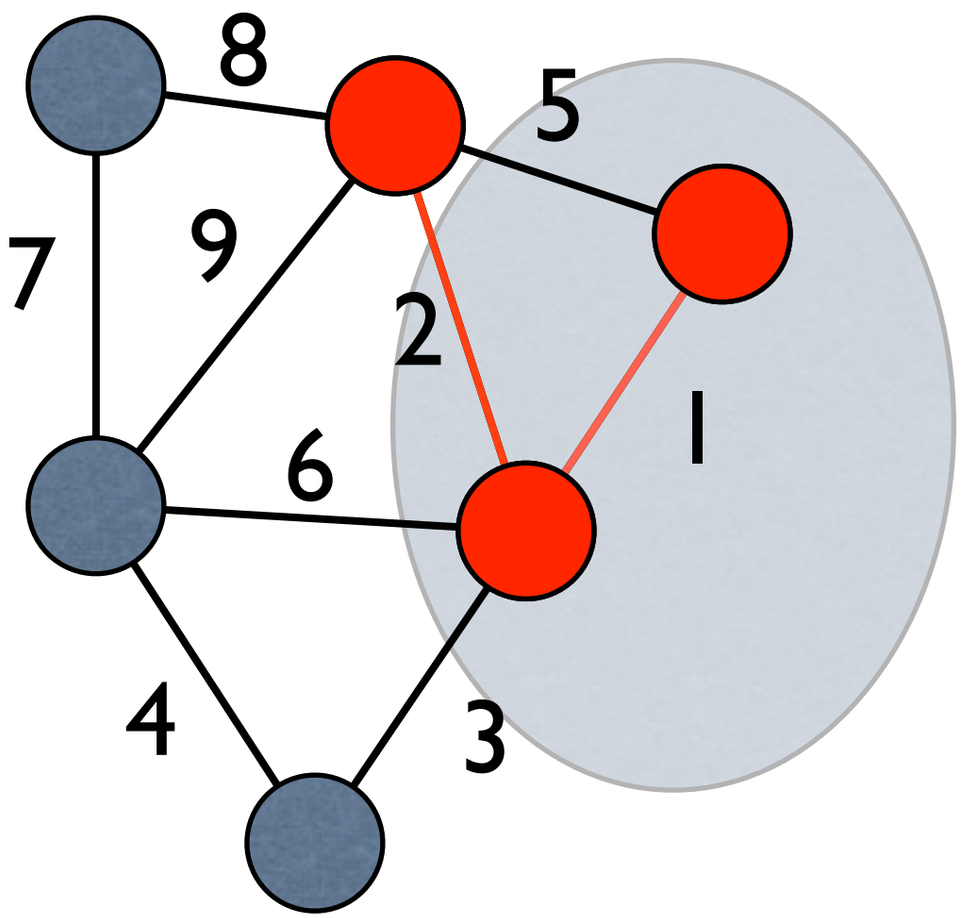

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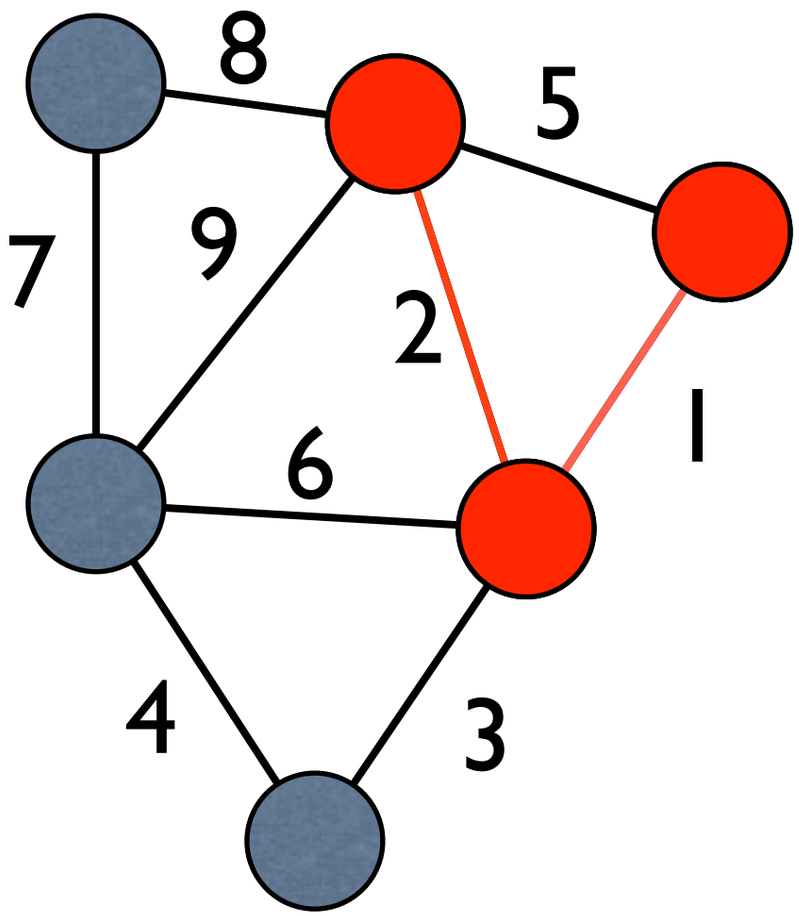

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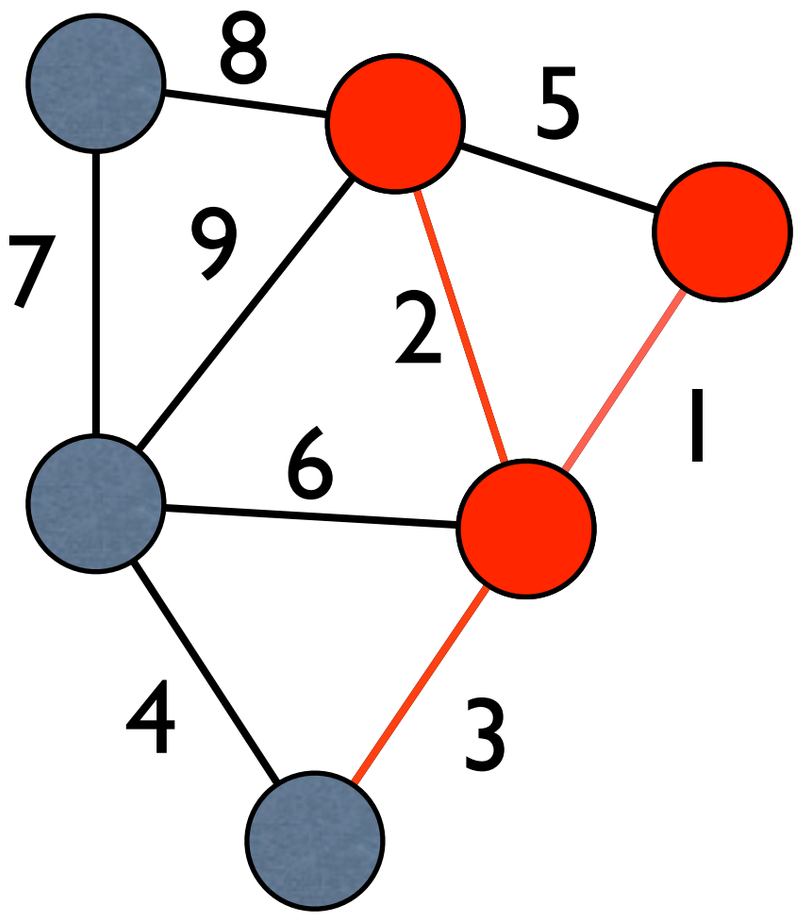

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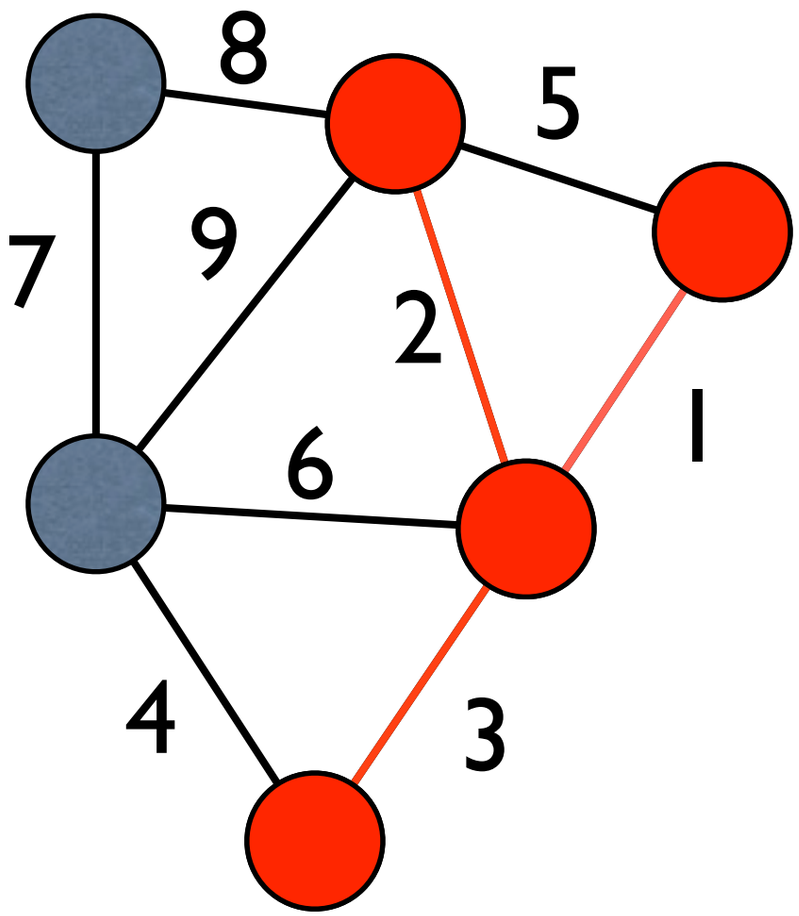

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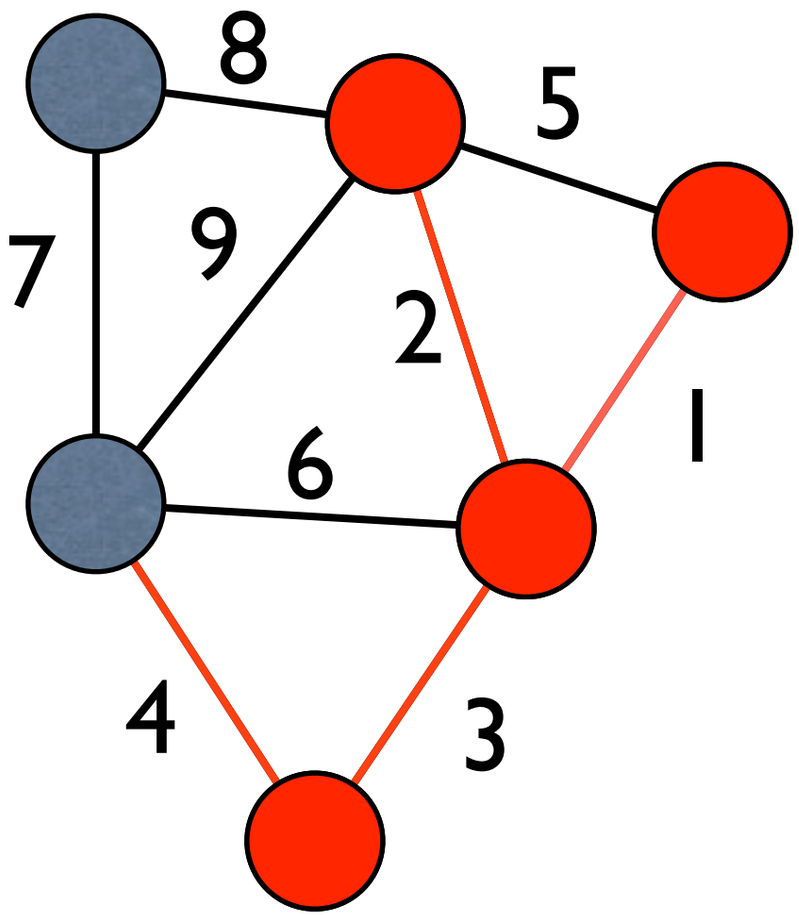

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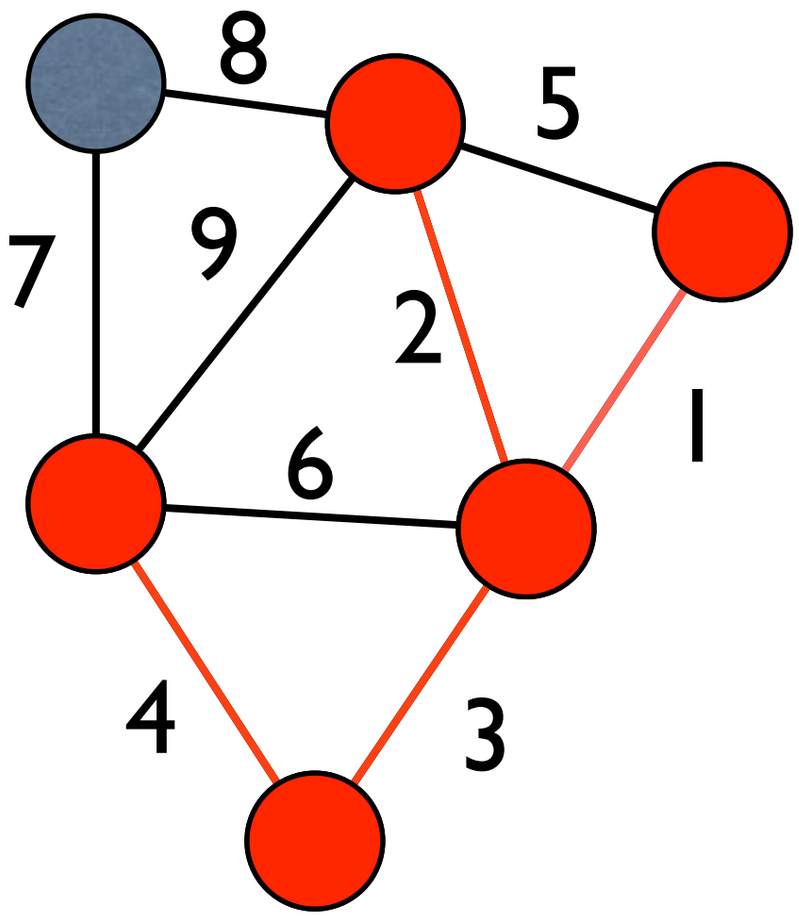

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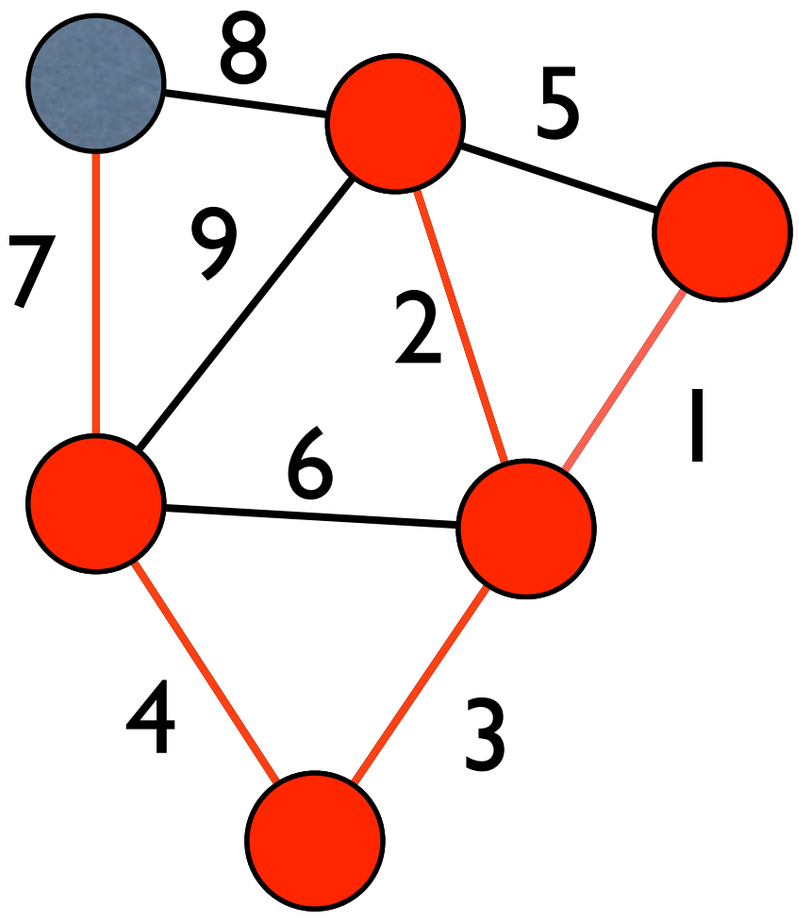

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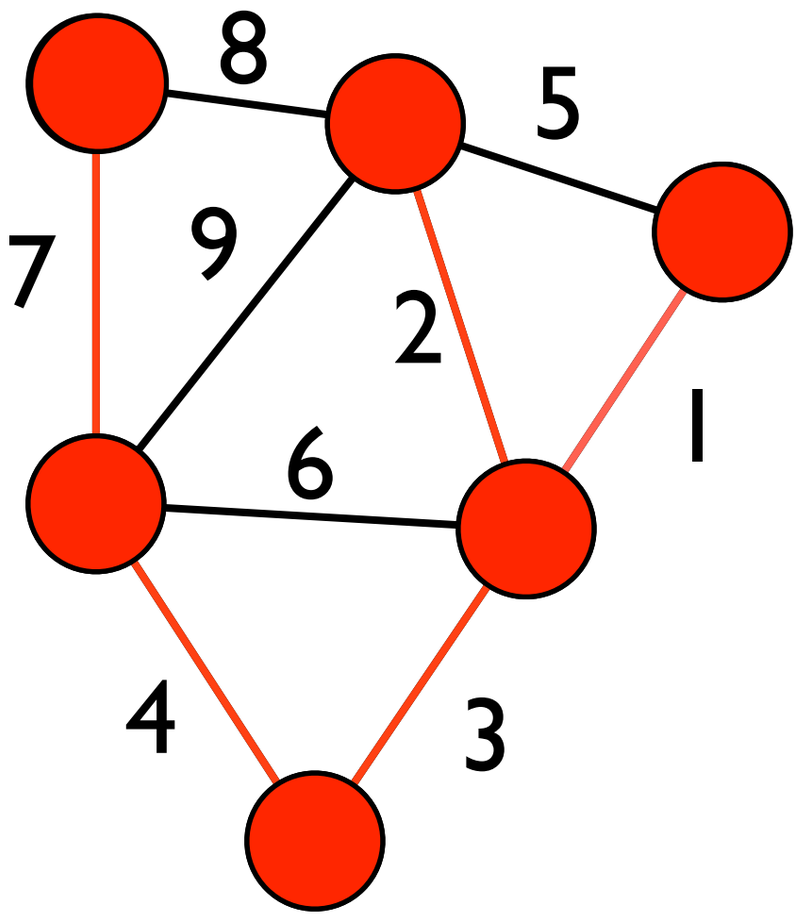
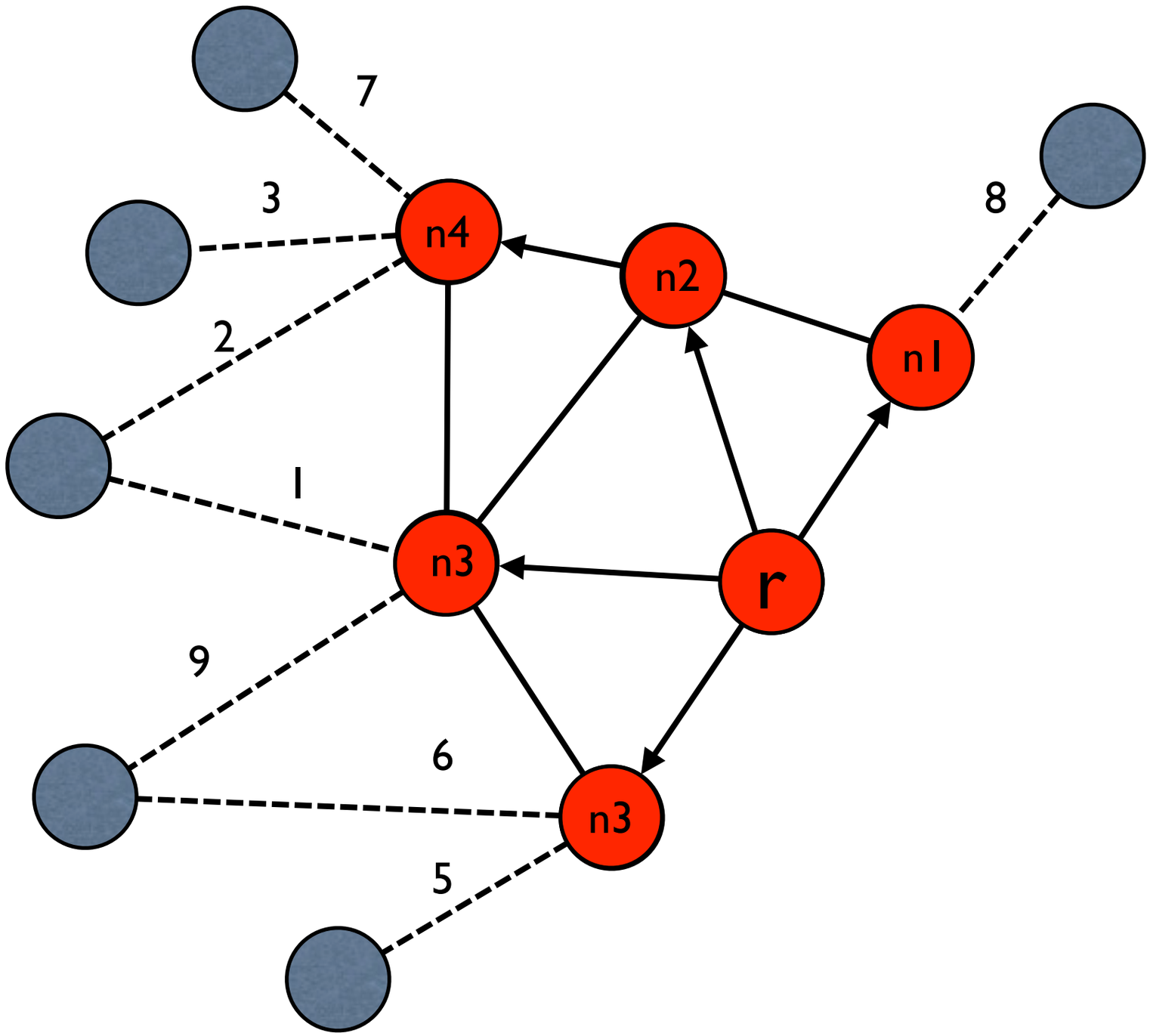


Distributed Prim

- root broadcasts «pulse» in the current tree
- upon reception of «pulse» each node of the current tree upcasts its lightest outgoing edge (not part of the current tree) to the root
- root chooses the lightest edge, e, and starts the next pulse (also broadcasting e)

Distributed Prim



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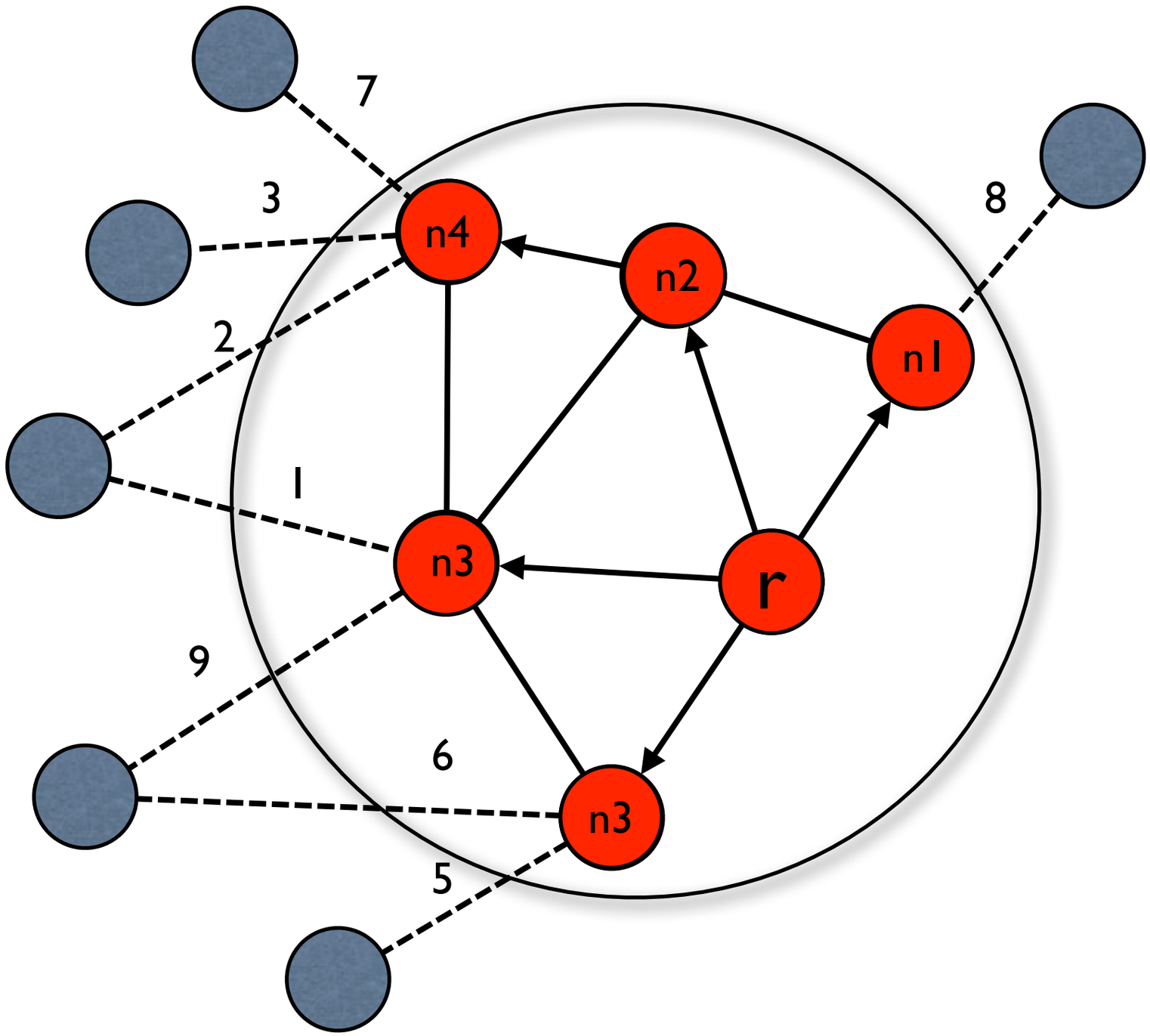

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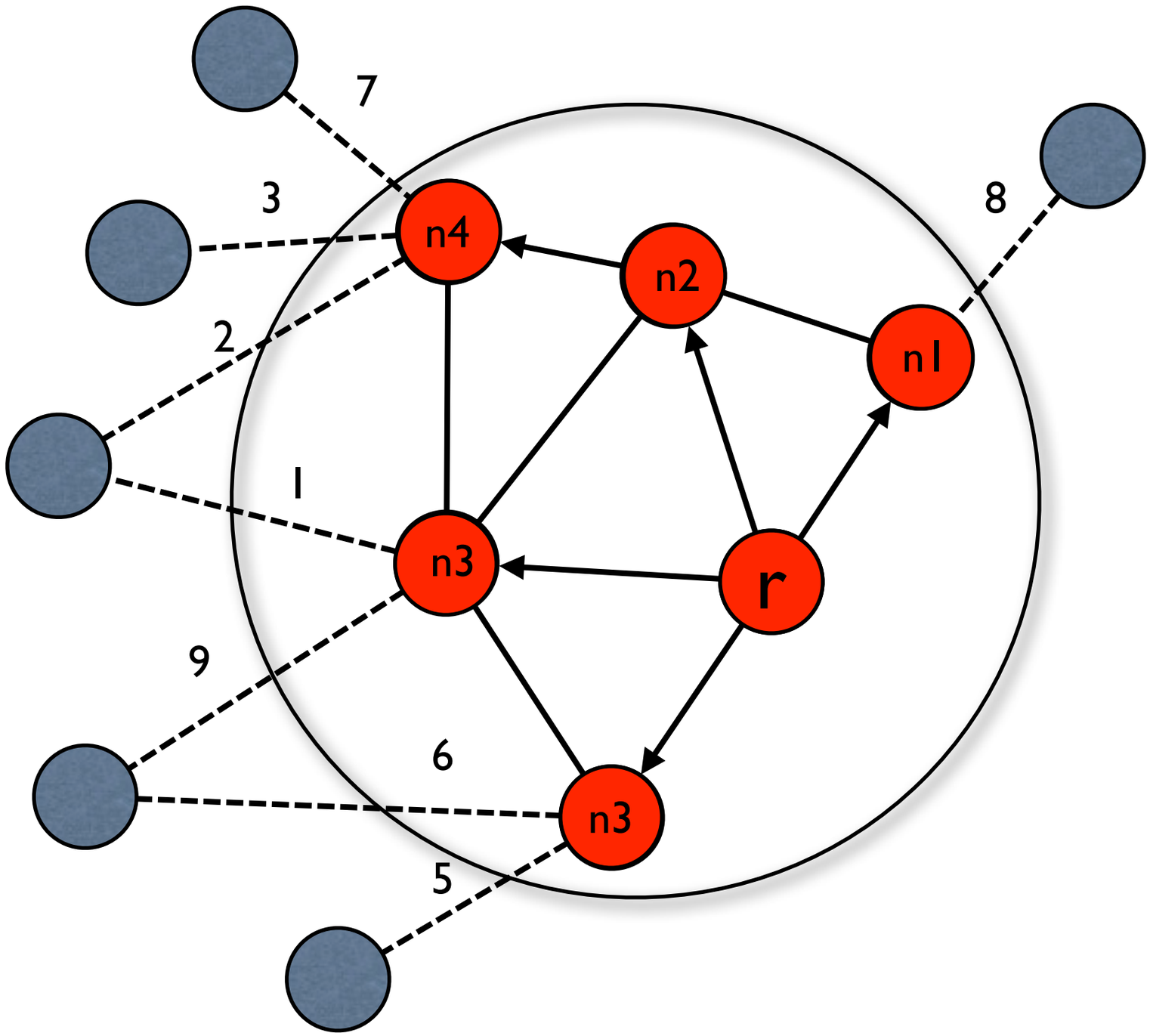

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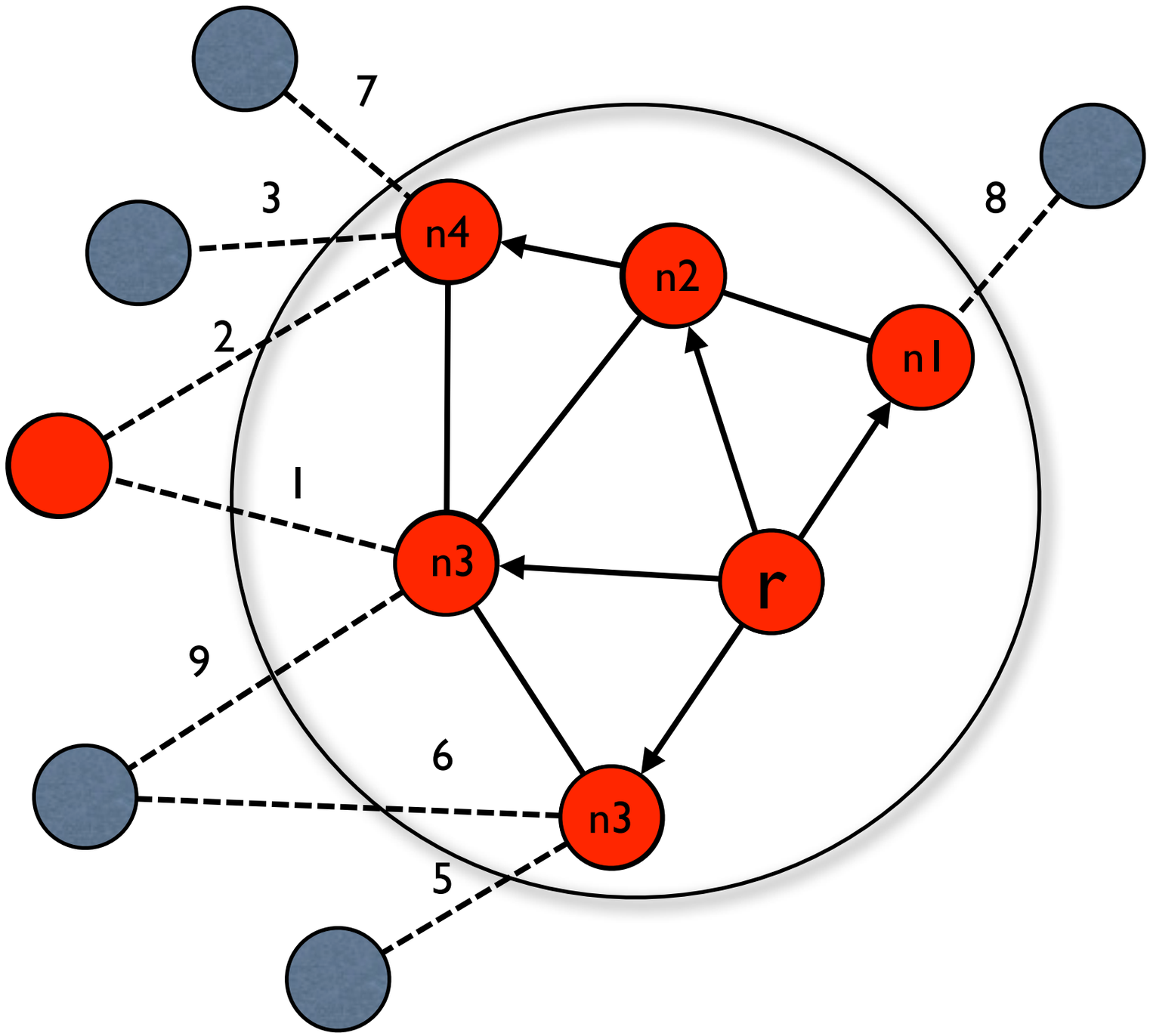

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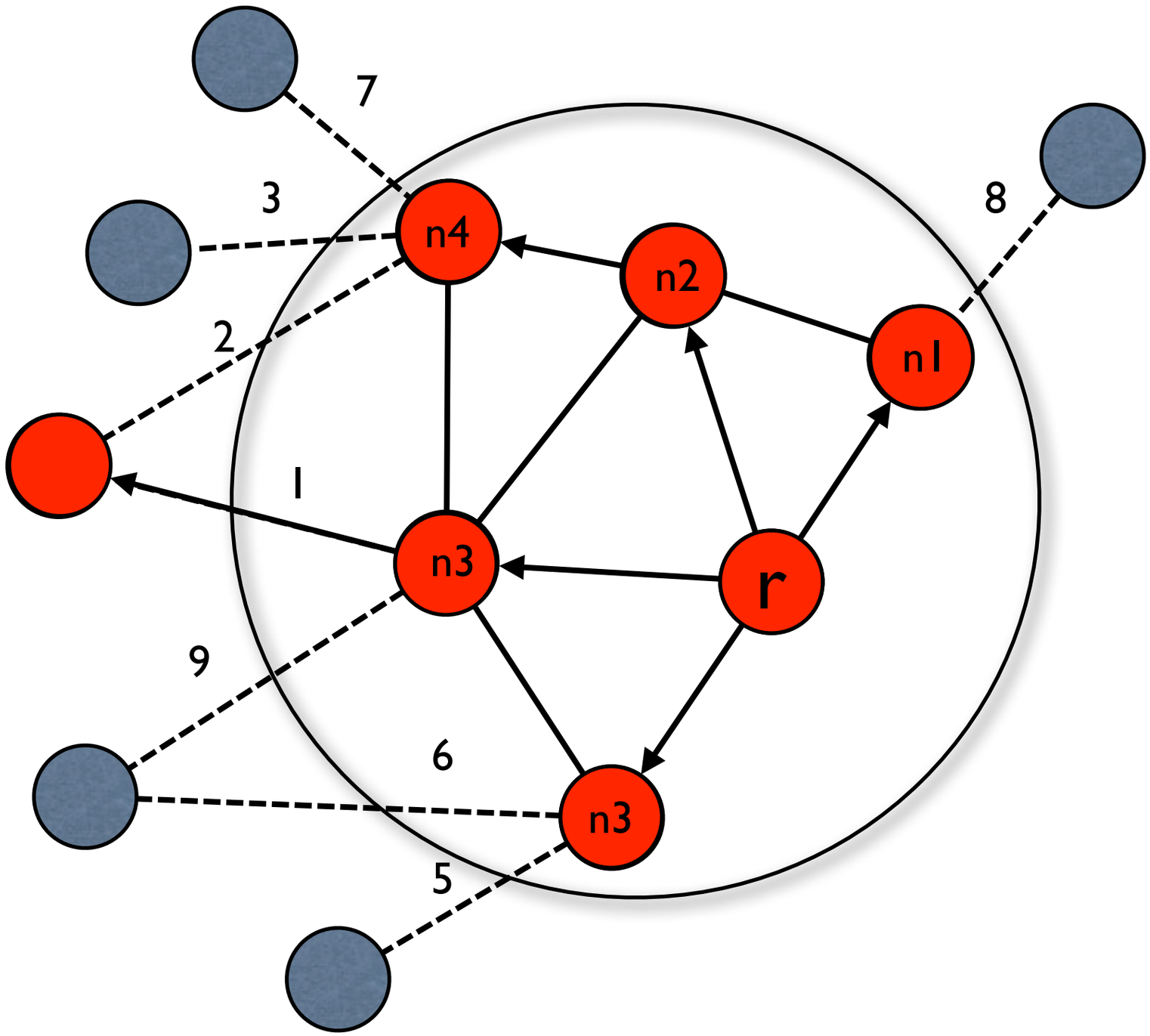

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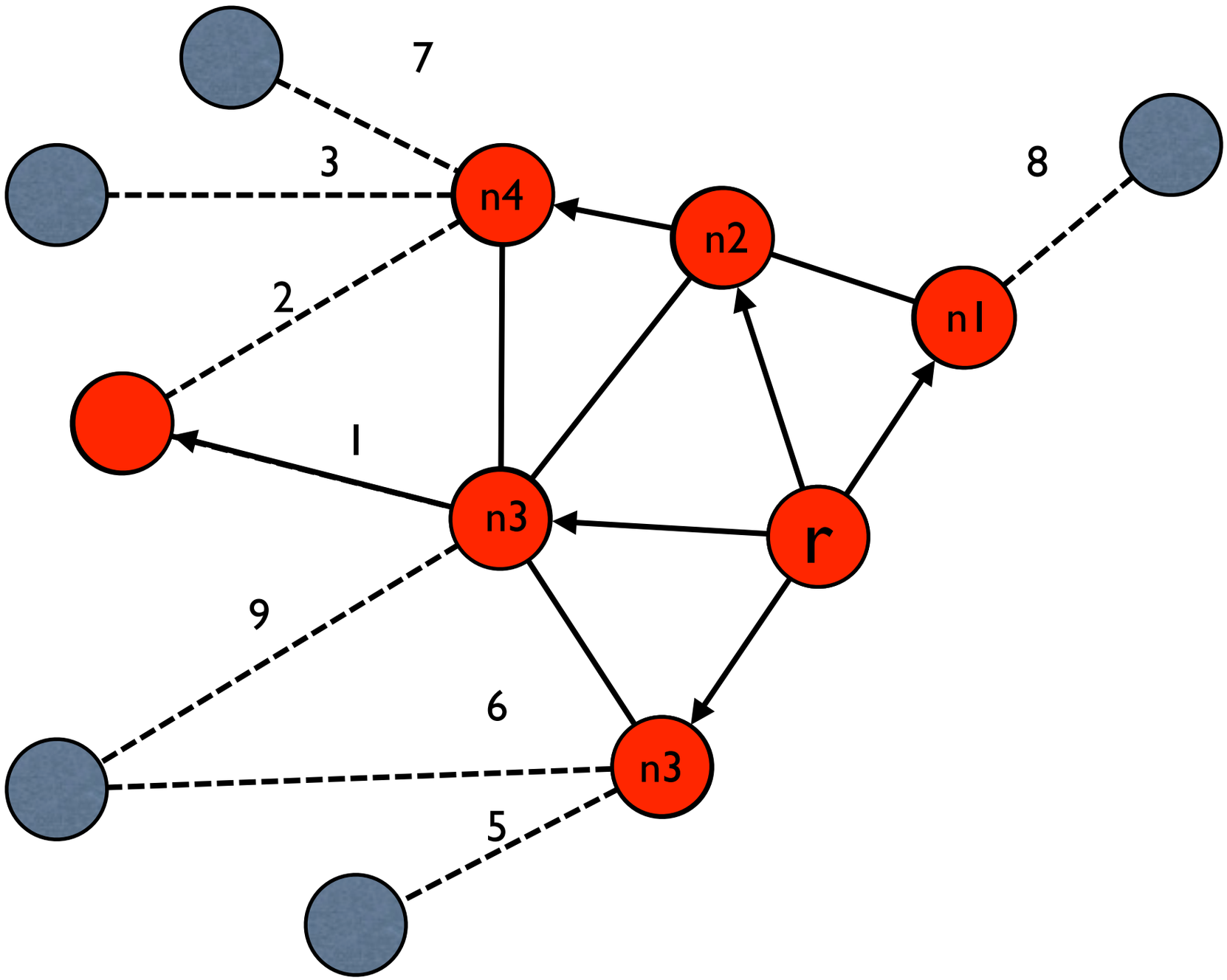

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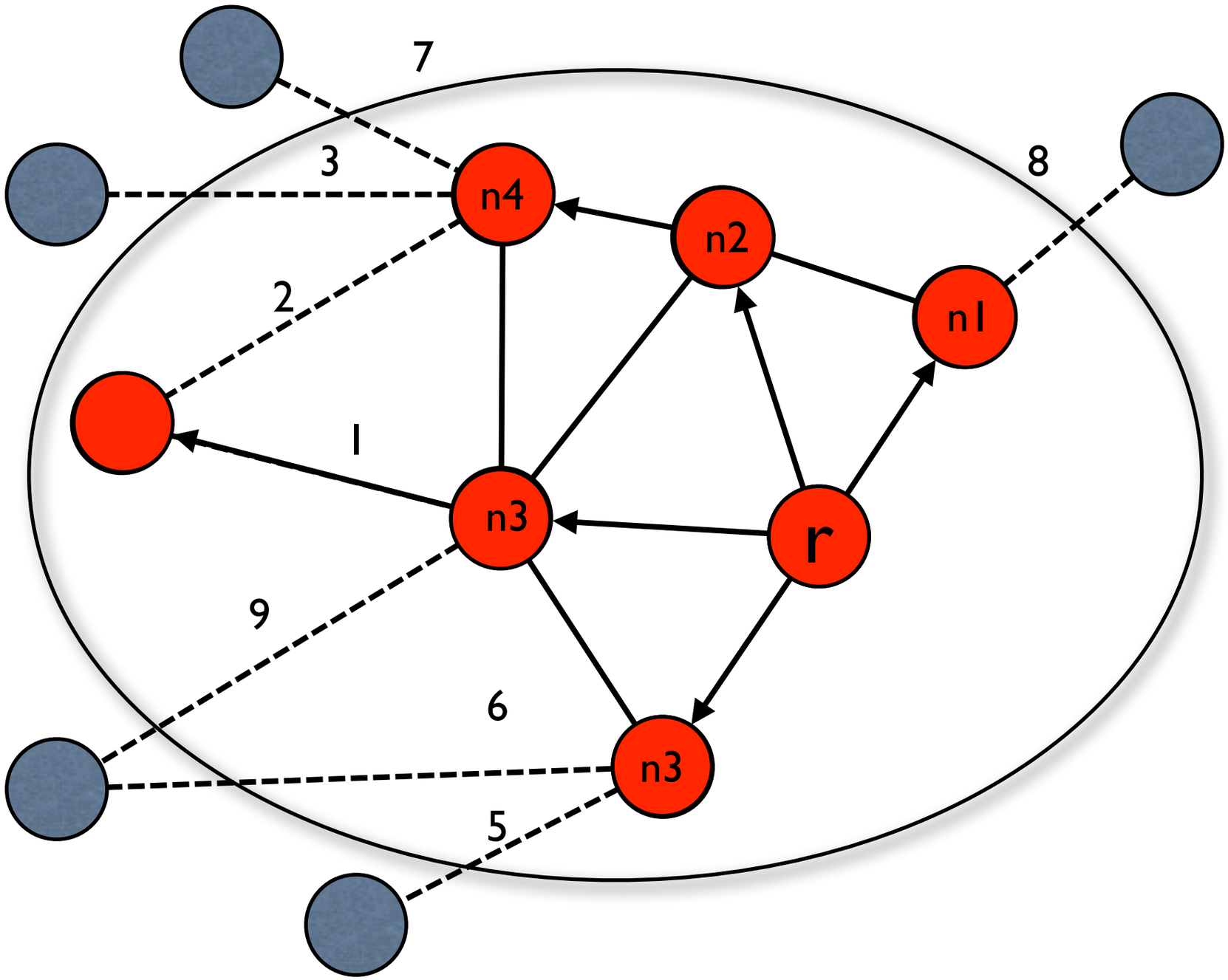
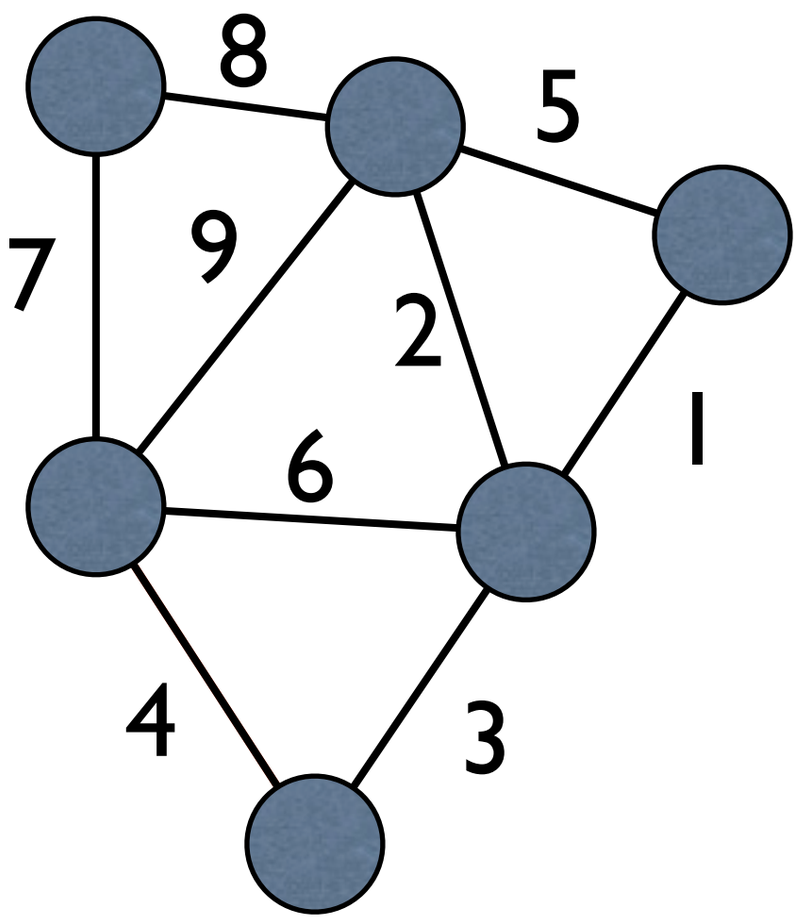


Distributed Prim - Complexity

- Message Complexity $O(n^2)$
- Time Complexity $O(n^2)$

Kruskal Algorithm

- each node is an initial fragment
- at each step apply the «blue» rule over all the fragments



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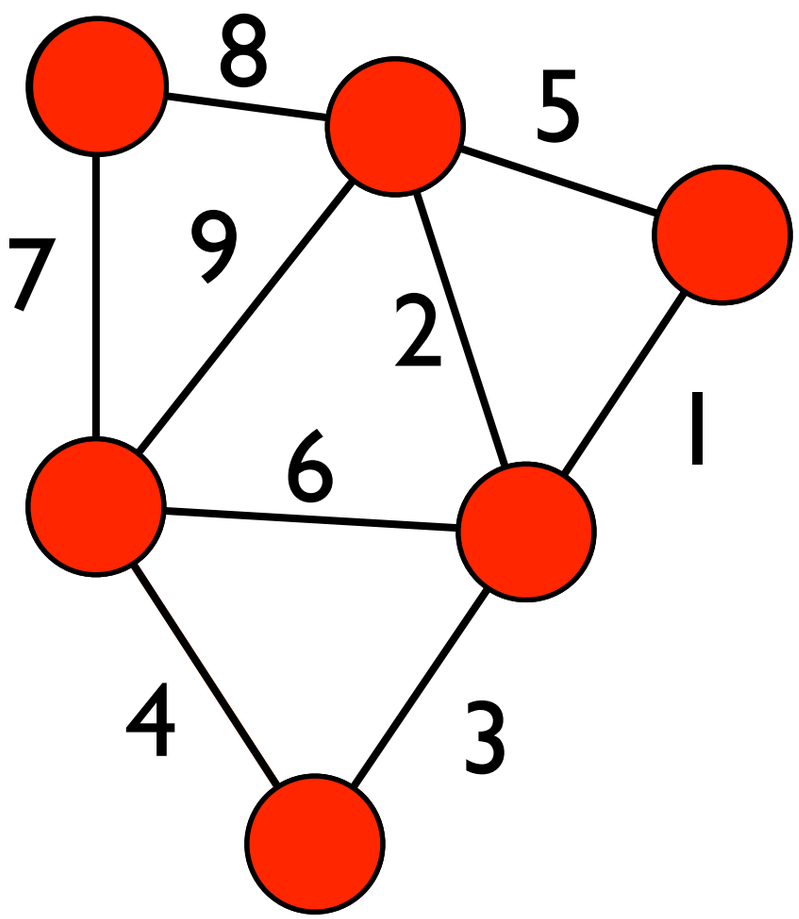

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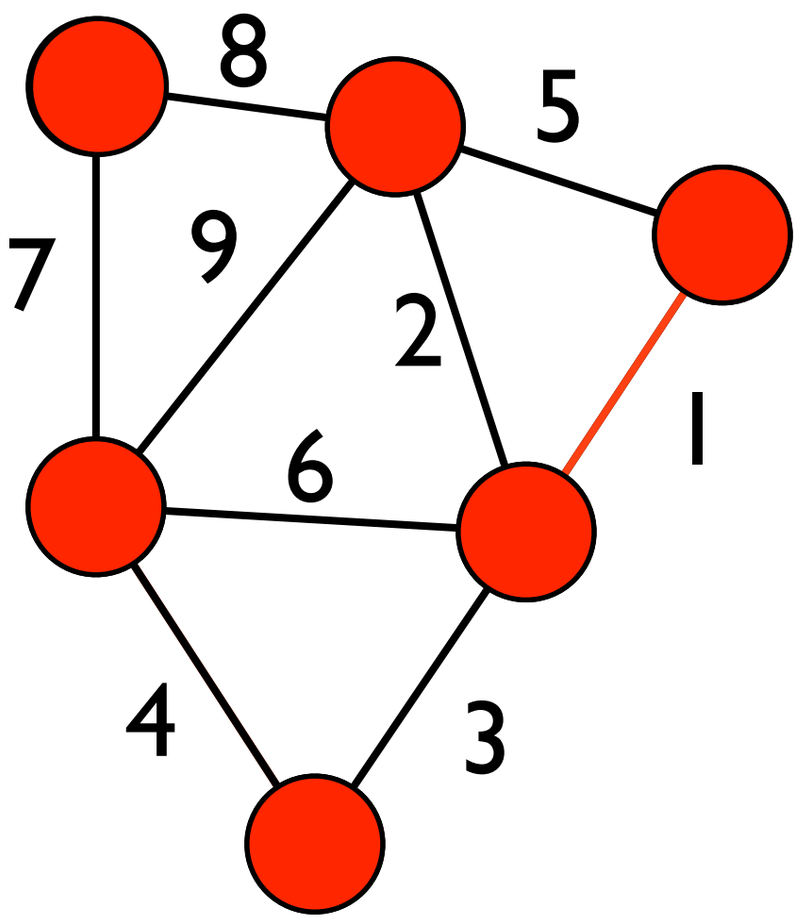

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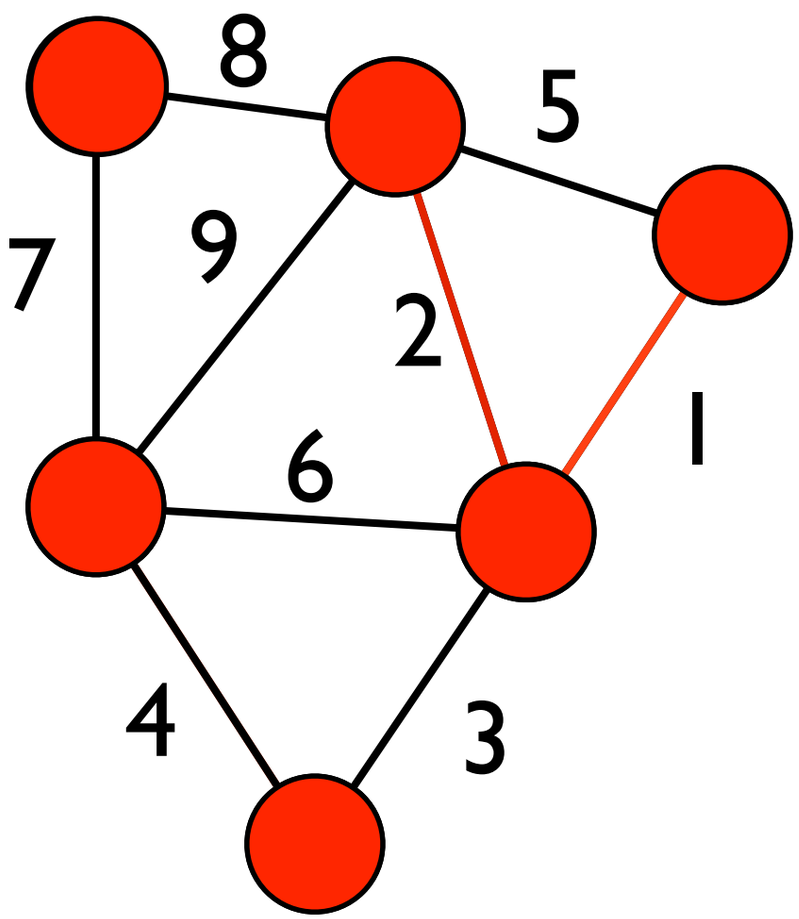

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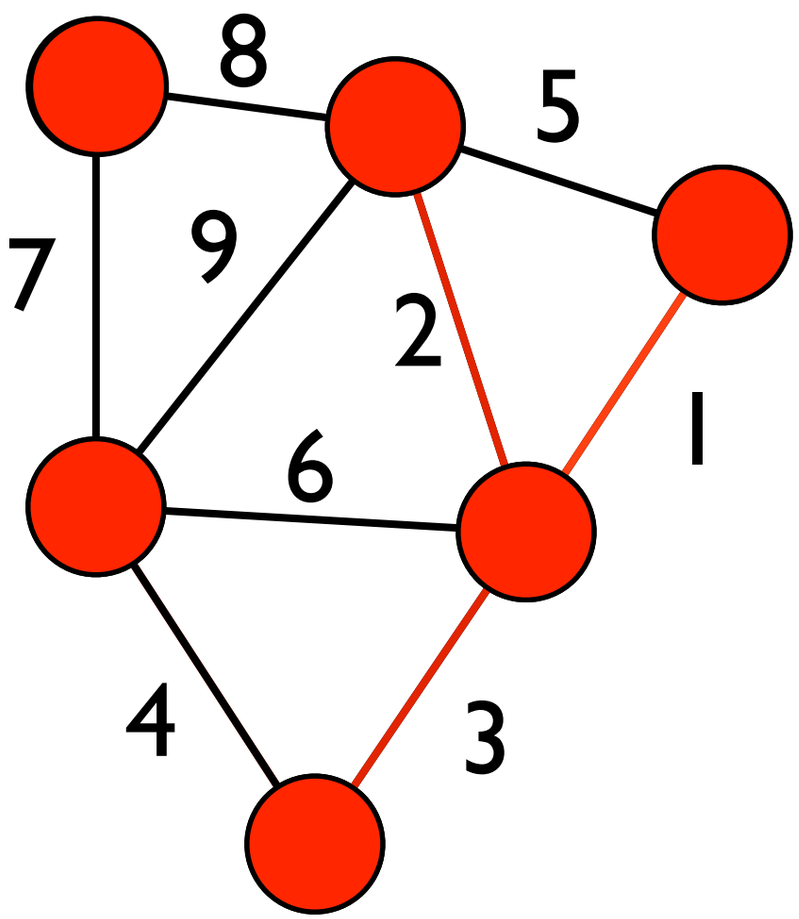

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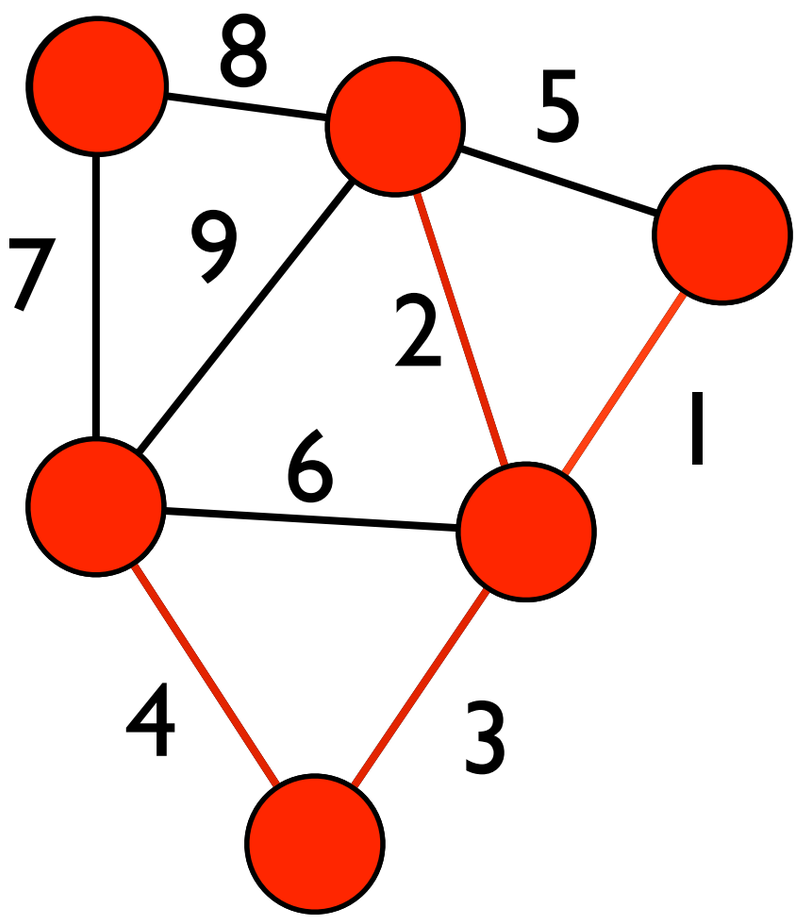

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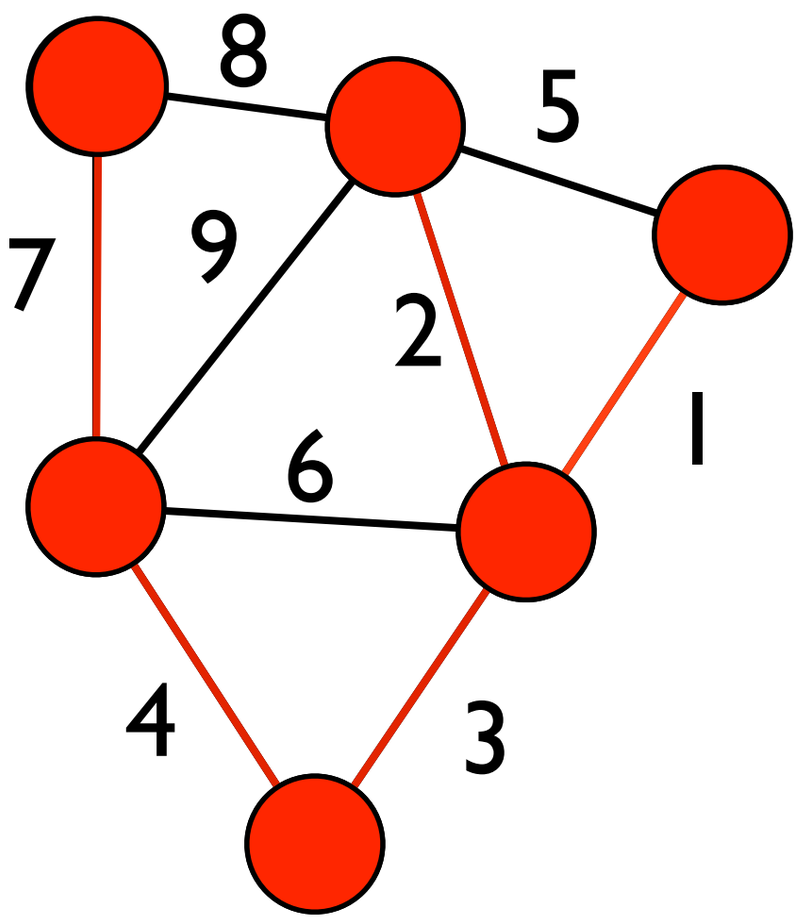

Distributed Kruskal (Gallager-Hamblet-Spira)

- Initially each node is the root of its fragment
- repeat until a unique fragment:

1. each root applies blue rule in its fragment
2. if (u, v)=mwoe(Tr) and (u,v)=mwoe(Ts) then u or v become the new root of the merged trees Tr and Ts (depending on the higher id)
3. new root informs the new fragment about its identifier

GHS Exemple

GHS - Complexity

- Message Complexity $O(|E|\log(n))$
- Time Complexity $O(n\log(n))$