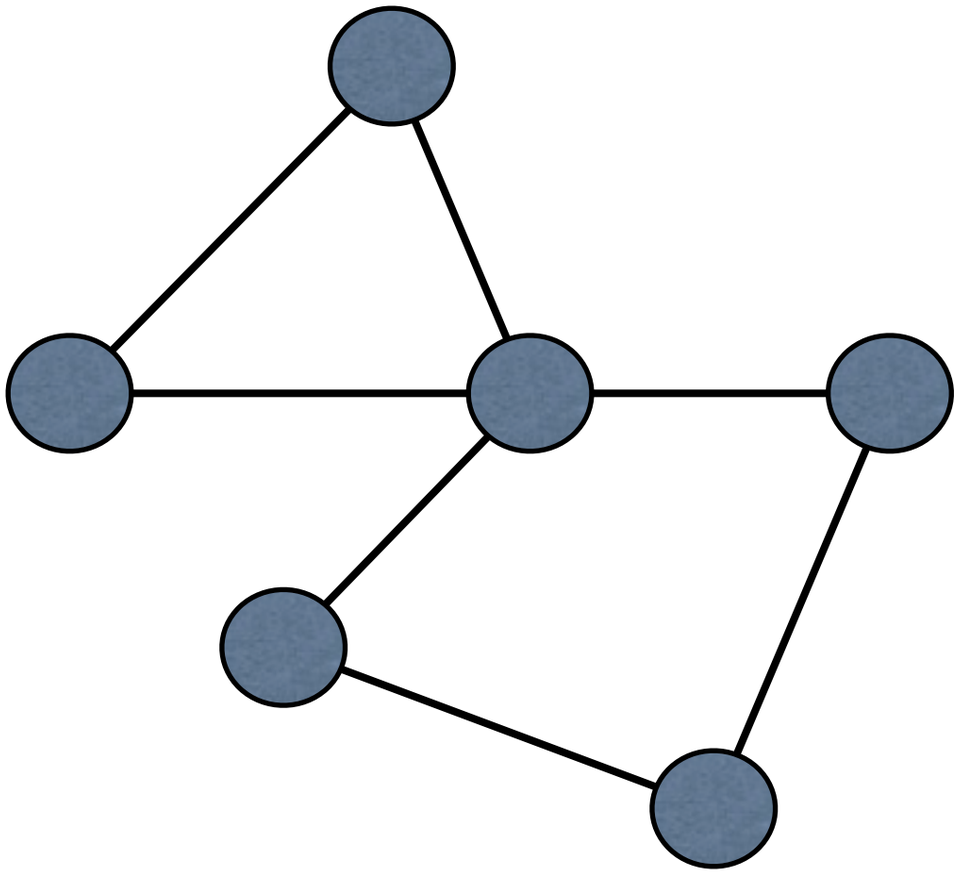
Independent and Dominating Sets

Why dominating or independent sets ?

Independent Sets

Independent Set

- Given a graph G=(V,E), an independent set is a subset of vertices, U, such that no two nodes in U are neighbors in G.



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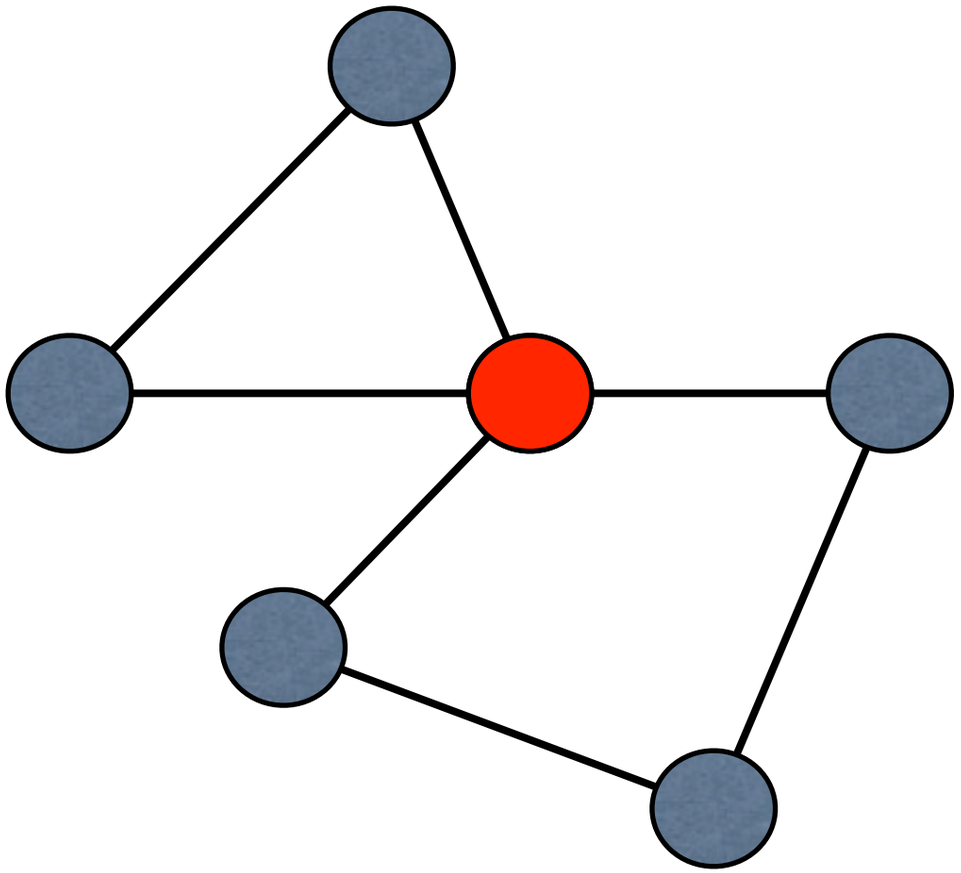
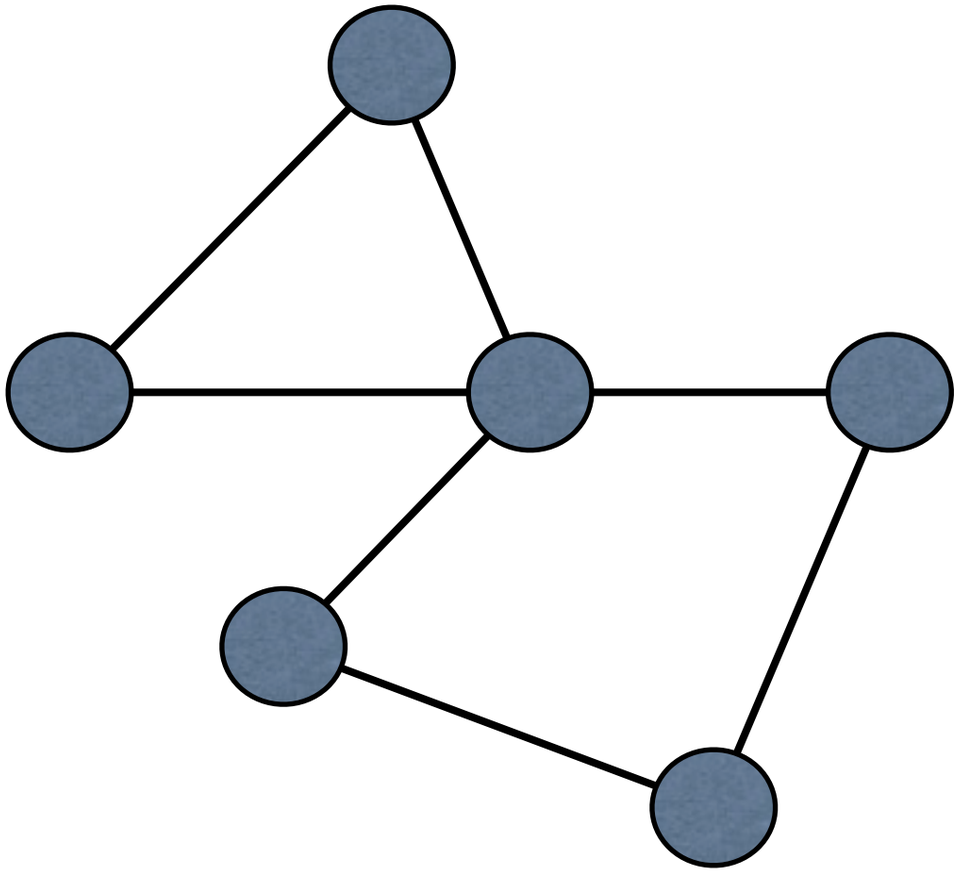


Maximal Independent Set (MIS)

- Given a graph G=(V,E), a **MIS** is a subset of vertices, U, such that :
  - **Independence:** no two nodes in U are neighbors in G
  - **Maximality:** no vertex can be added to U without violating the independence



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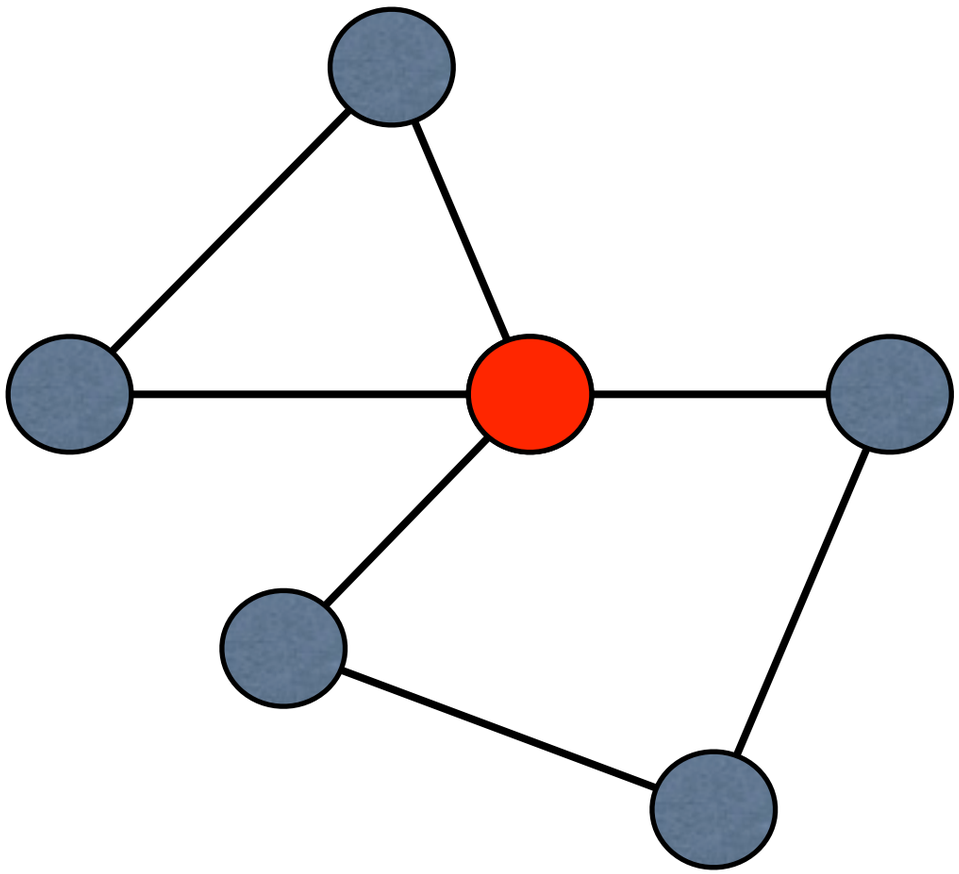

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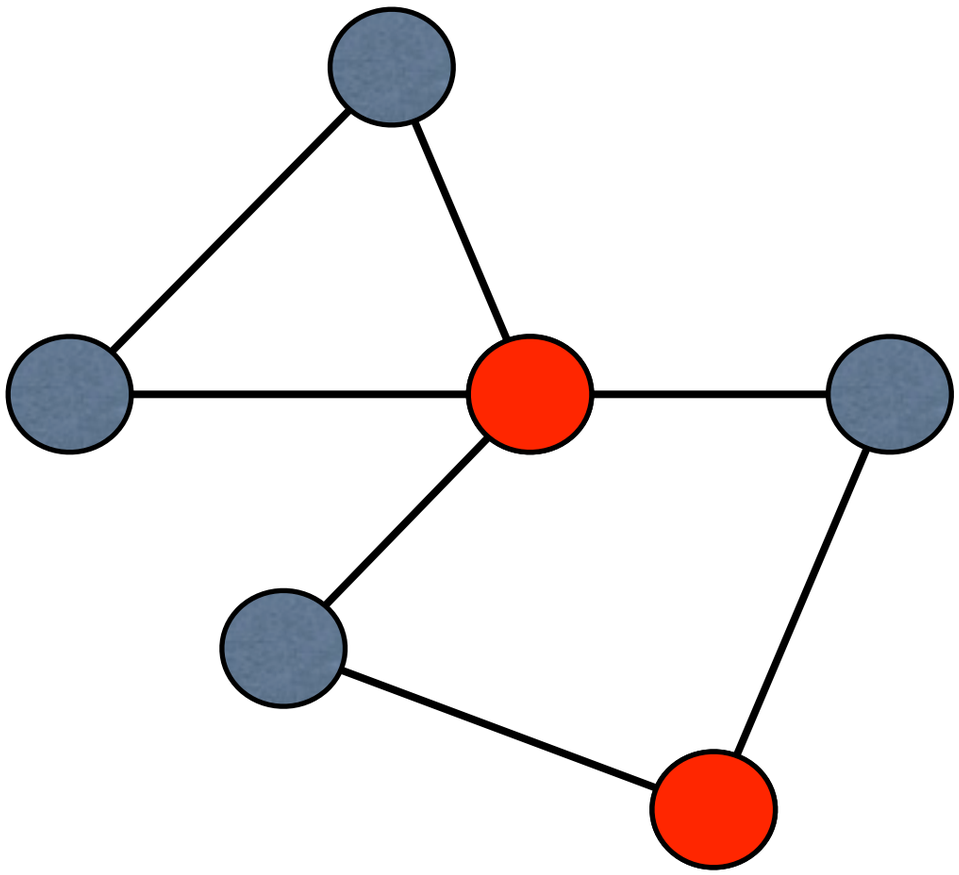
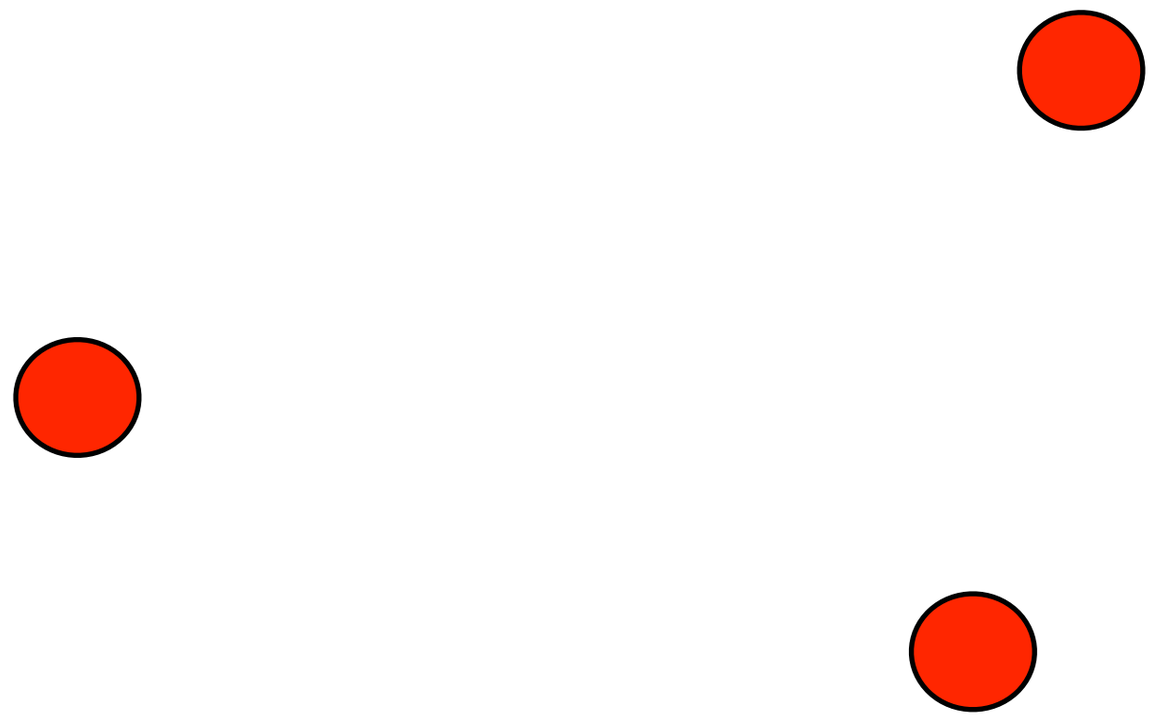


Maximal Independent Sets ?



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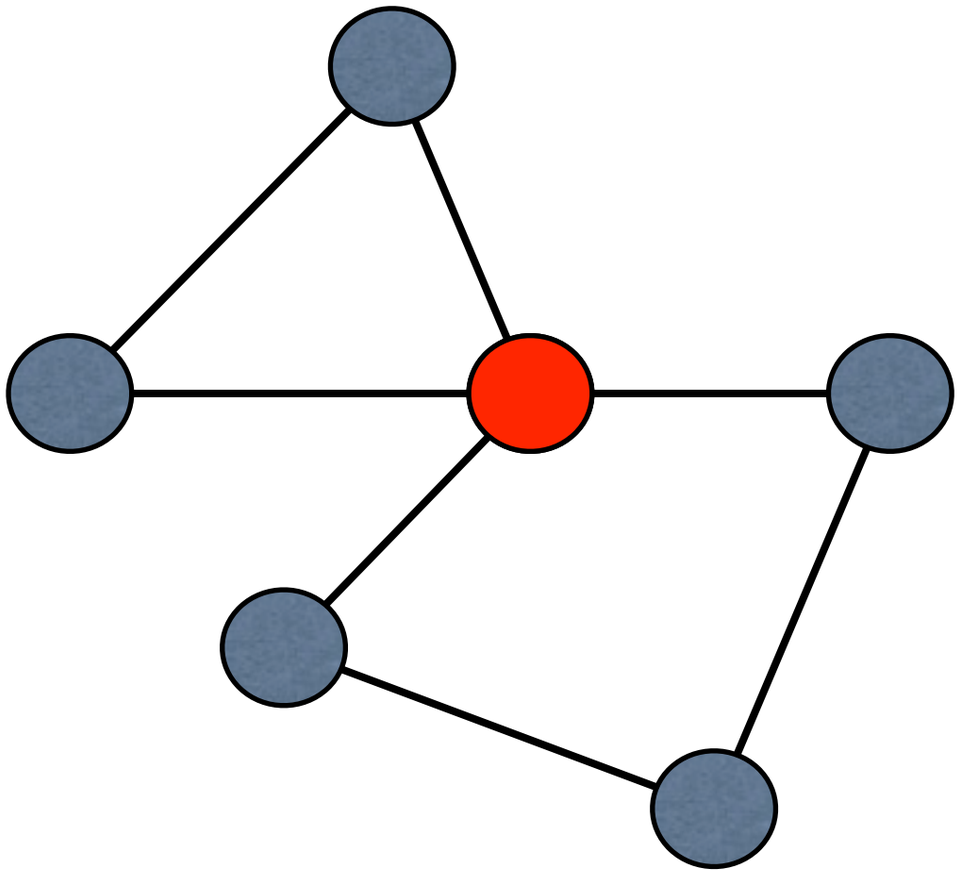

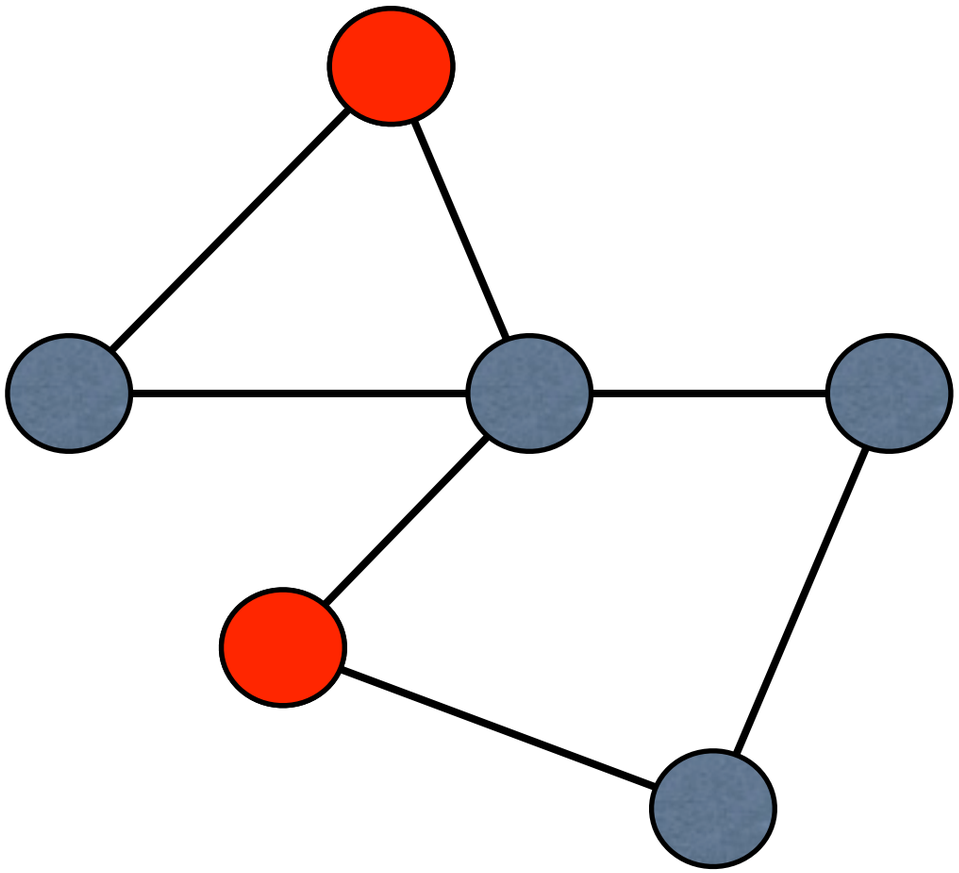

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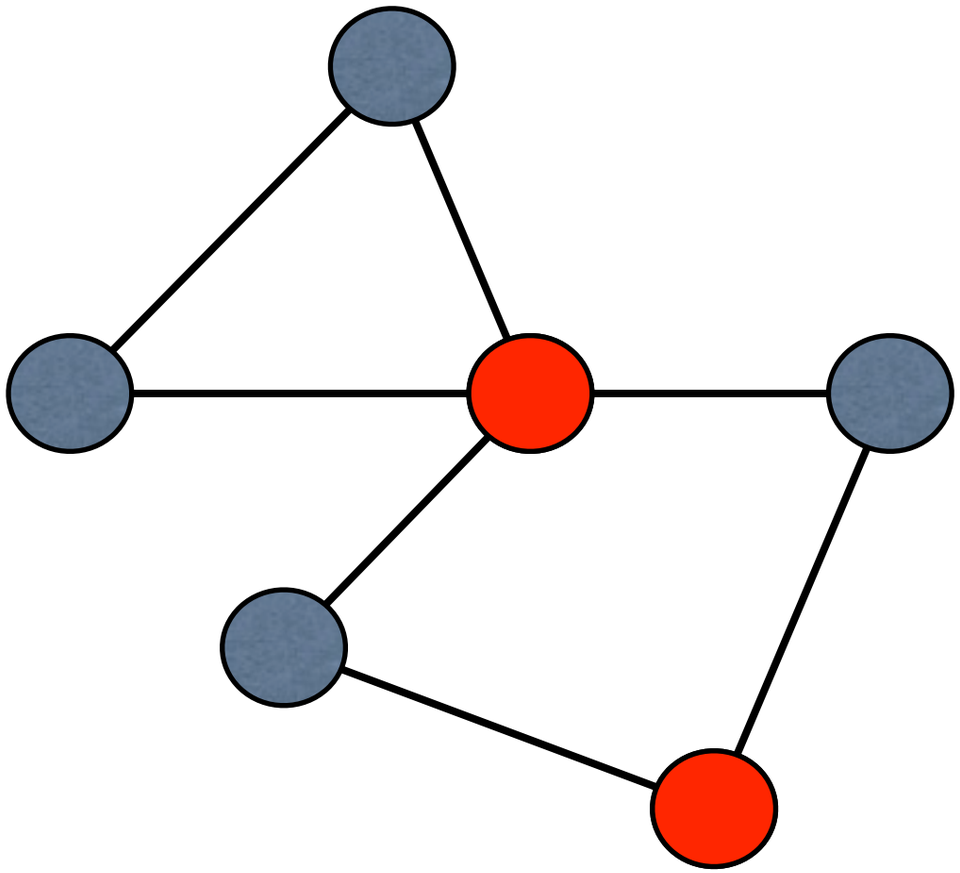
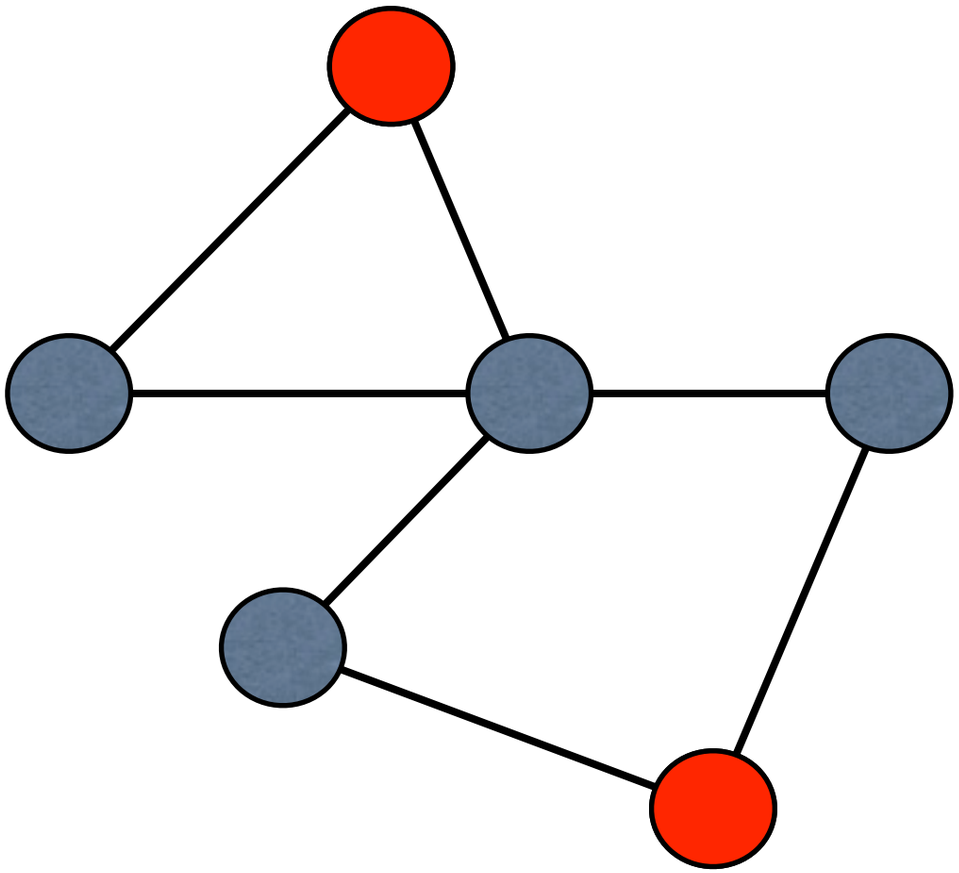
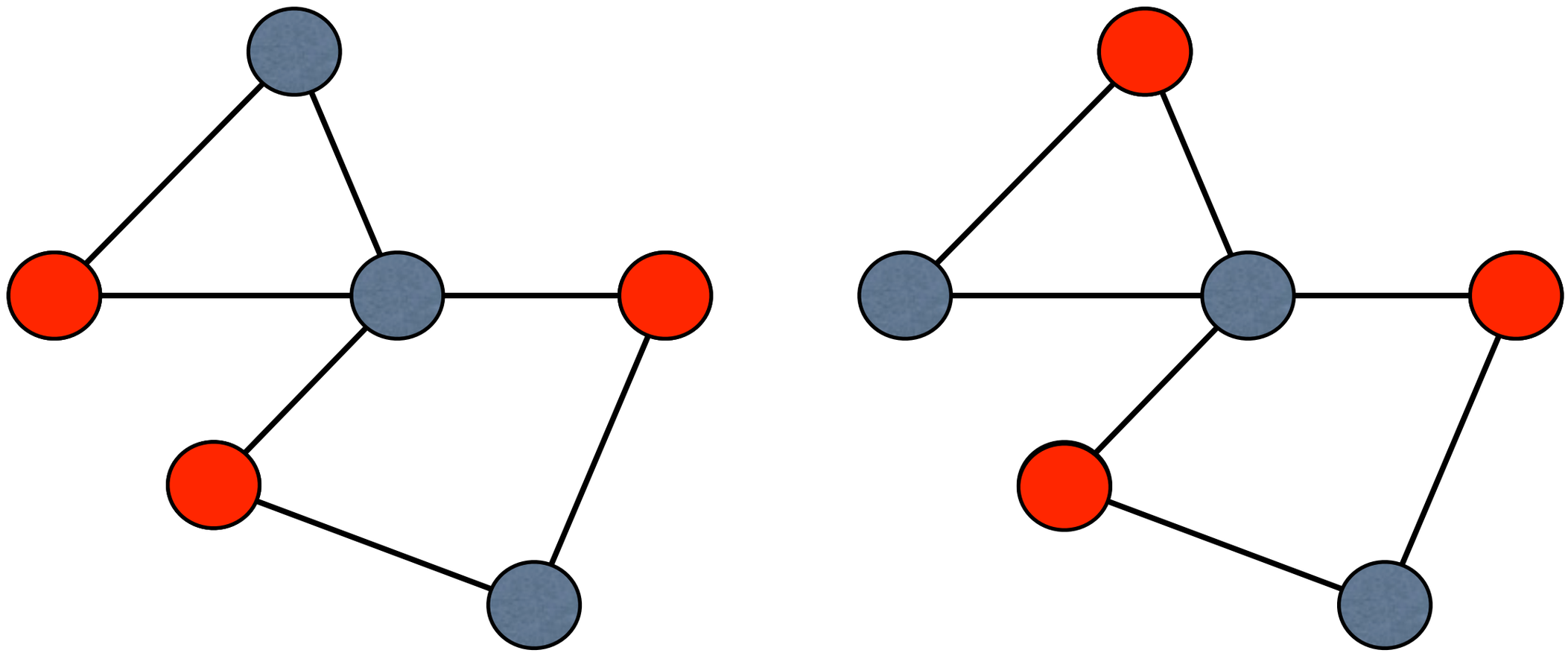


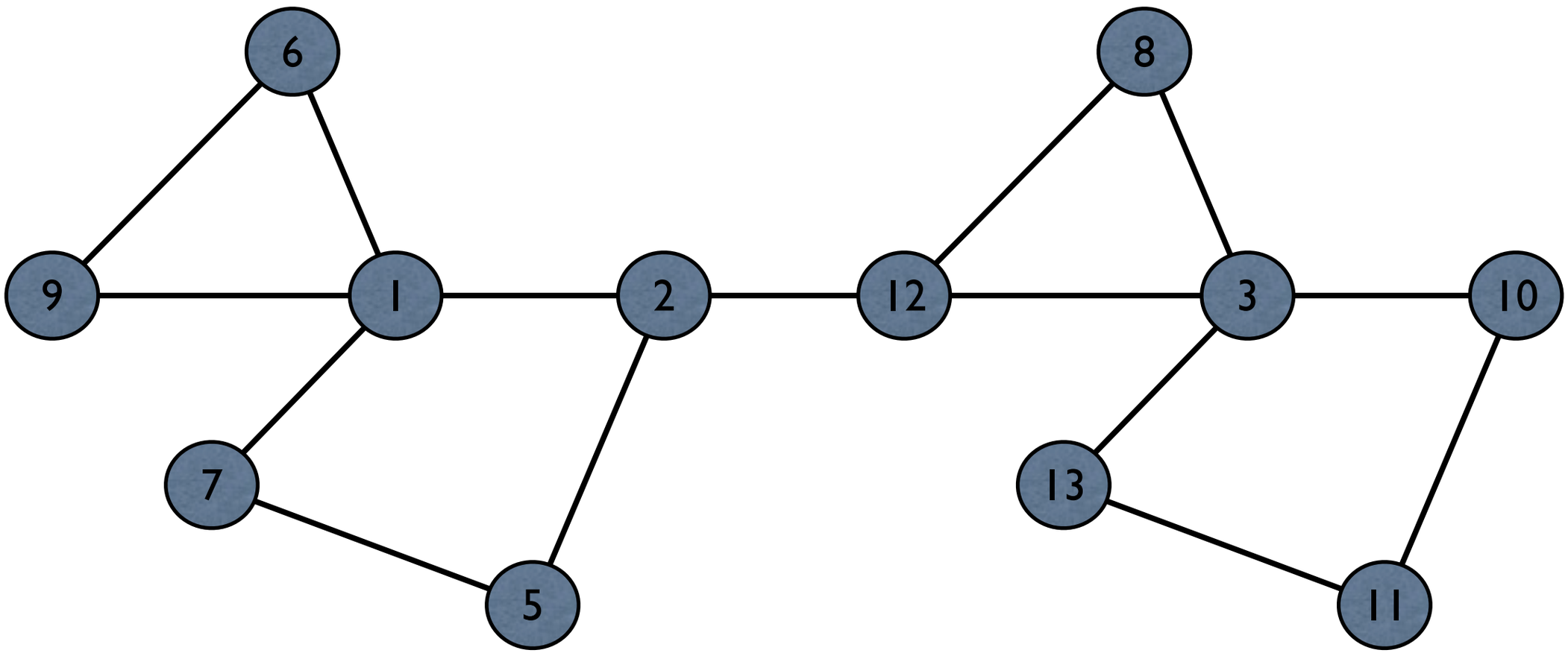


Maximal Independent Sets ?

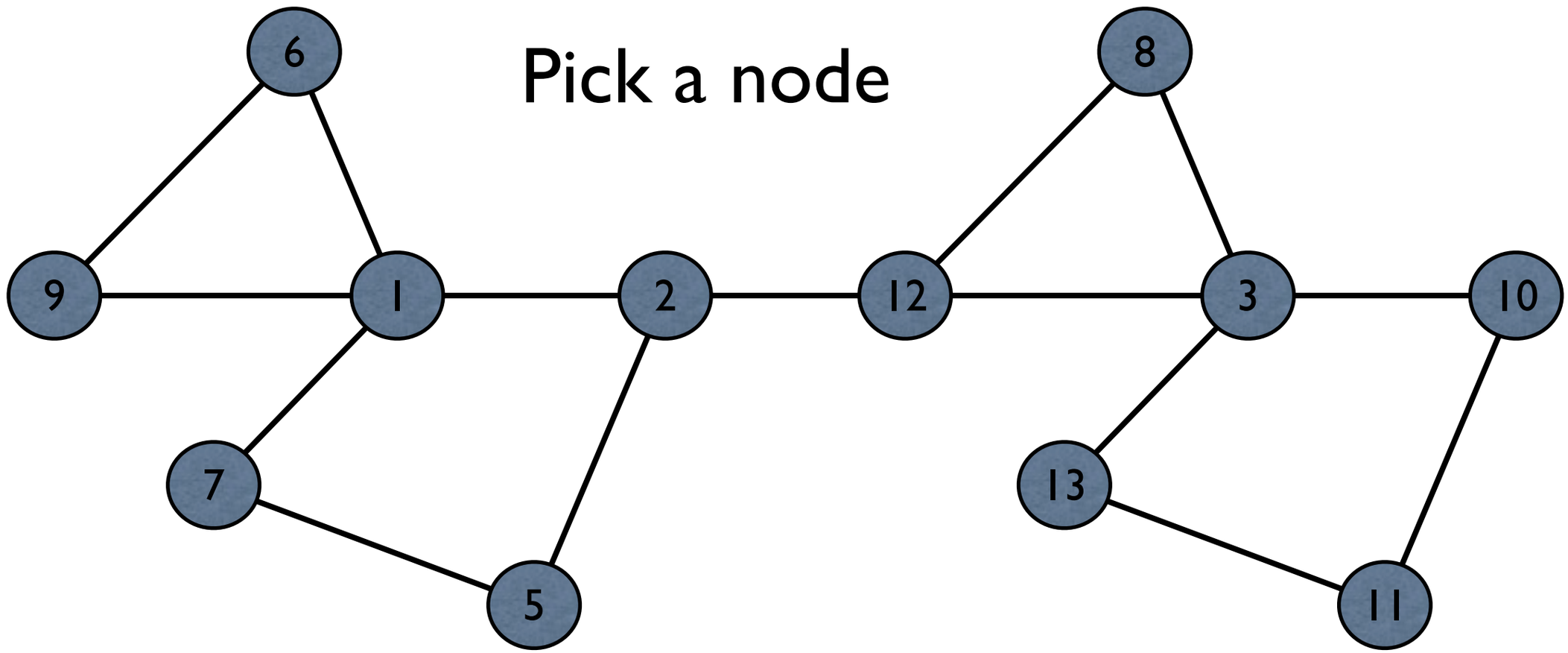


MIS Implementation

Centralized MIS



Centralized MIS



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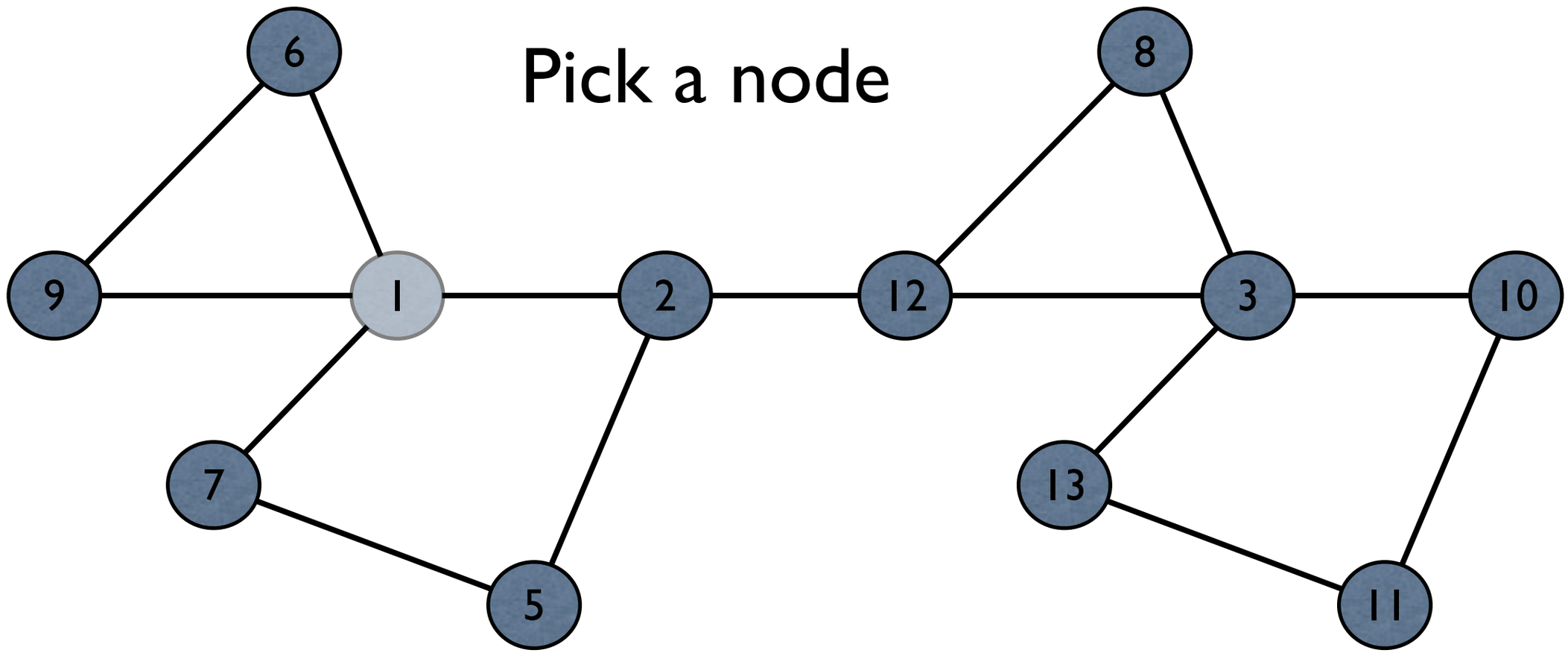

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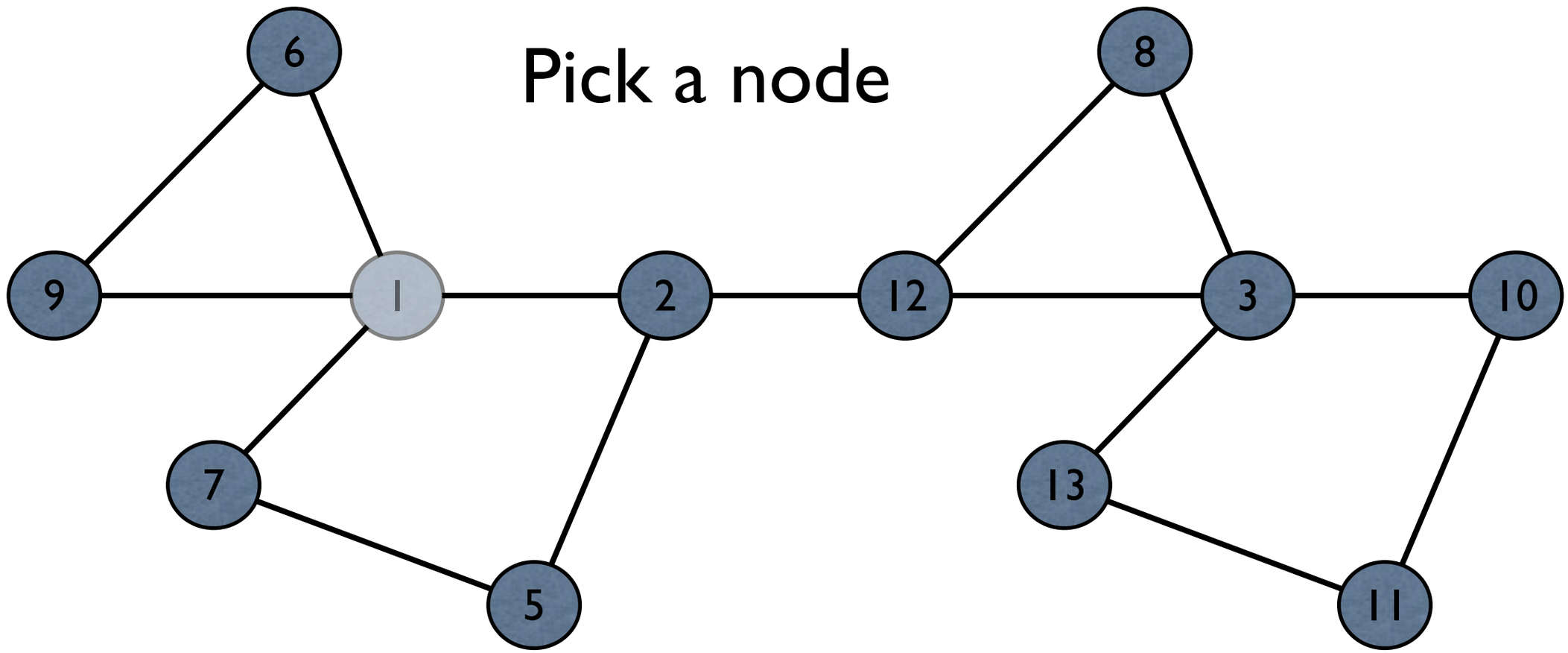

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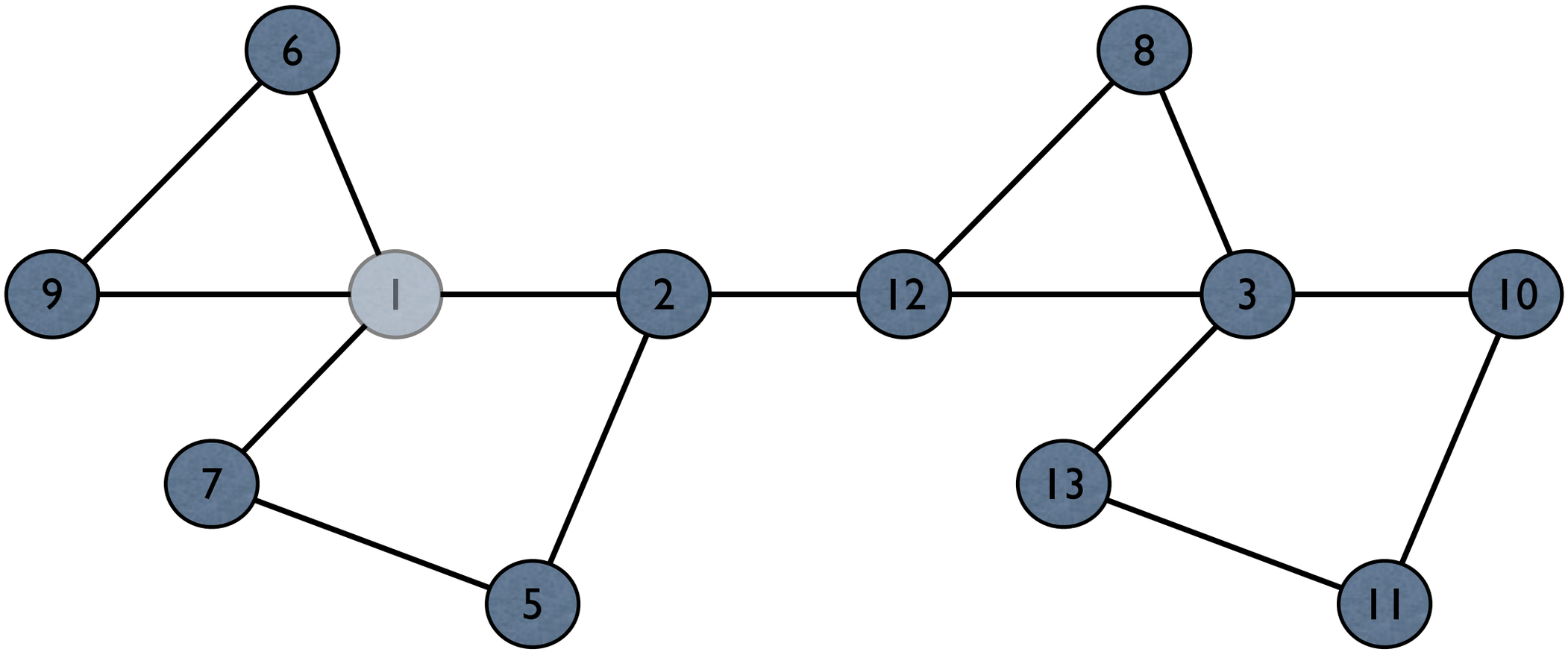
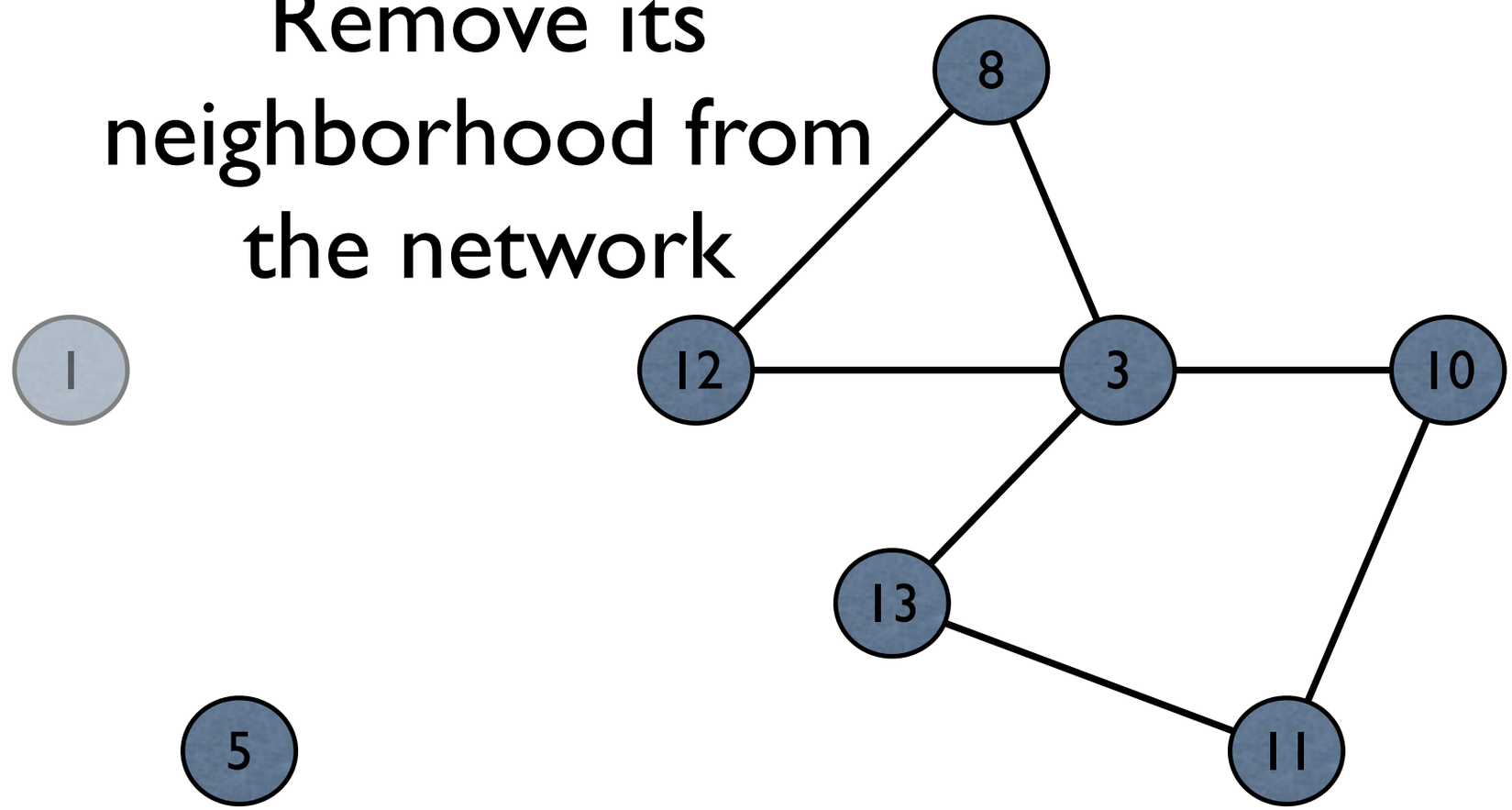


Centralized MIS

Remove its neighborhood from the network



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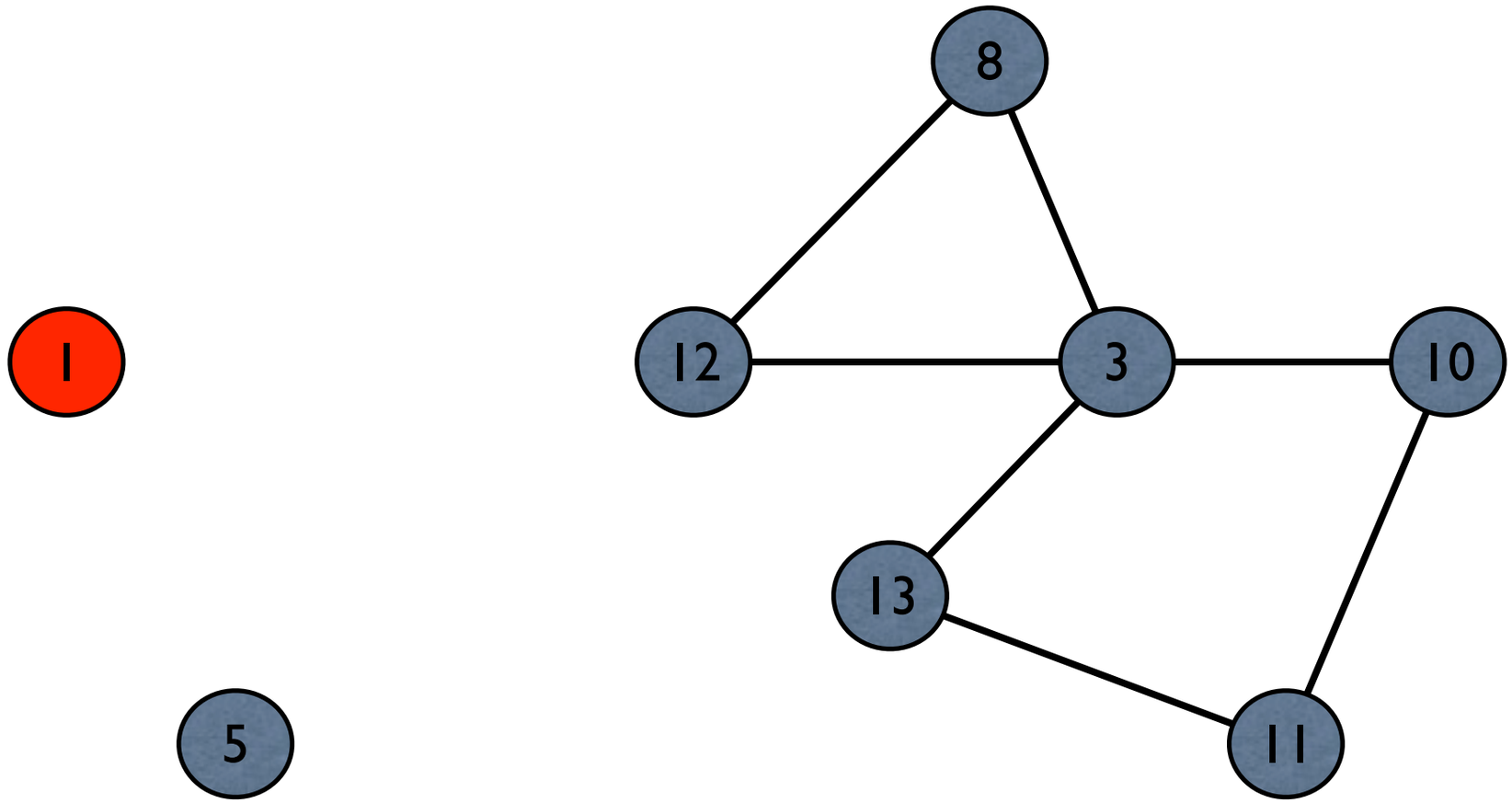
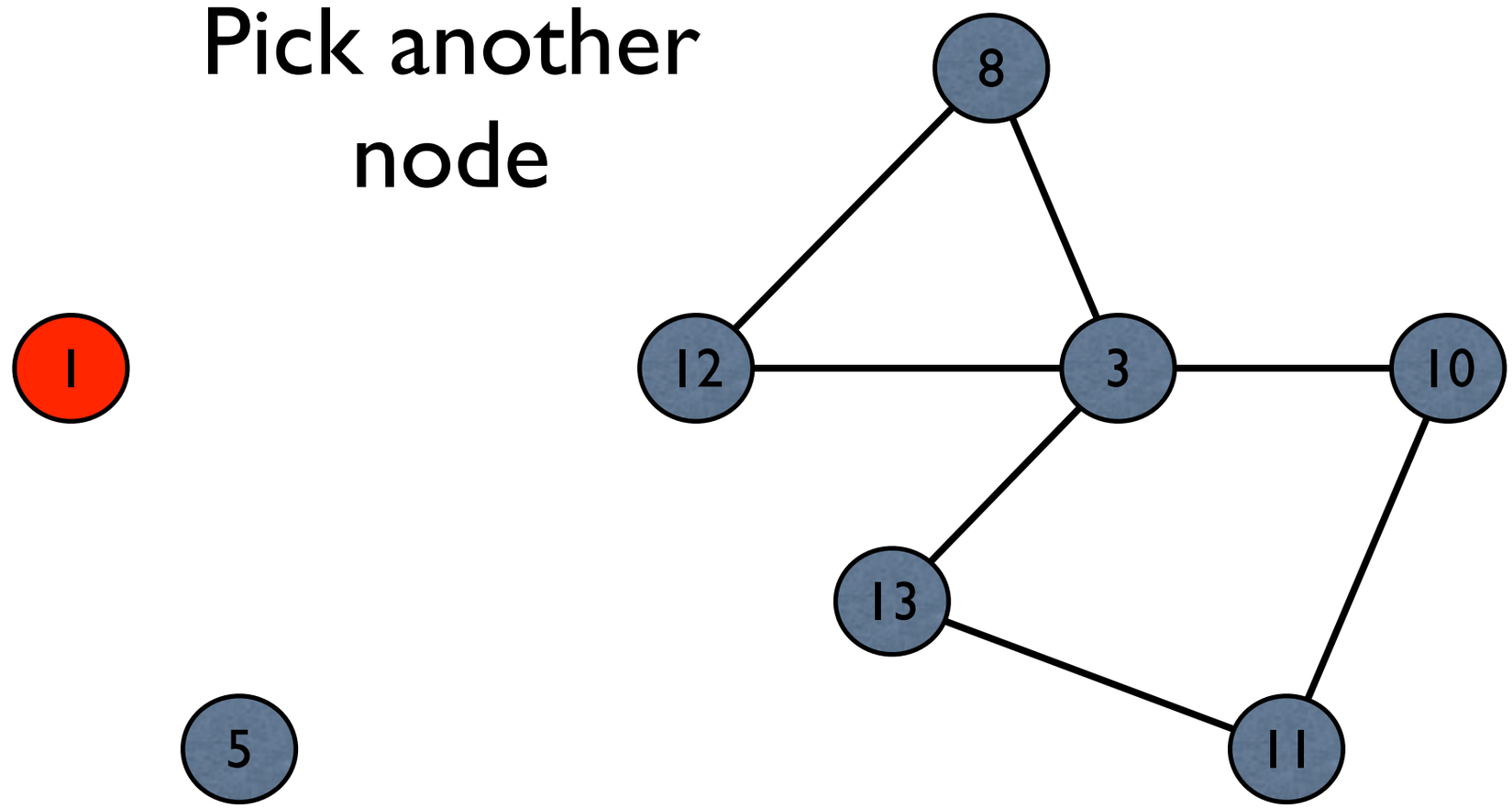


Centralized MIS

Pick another node



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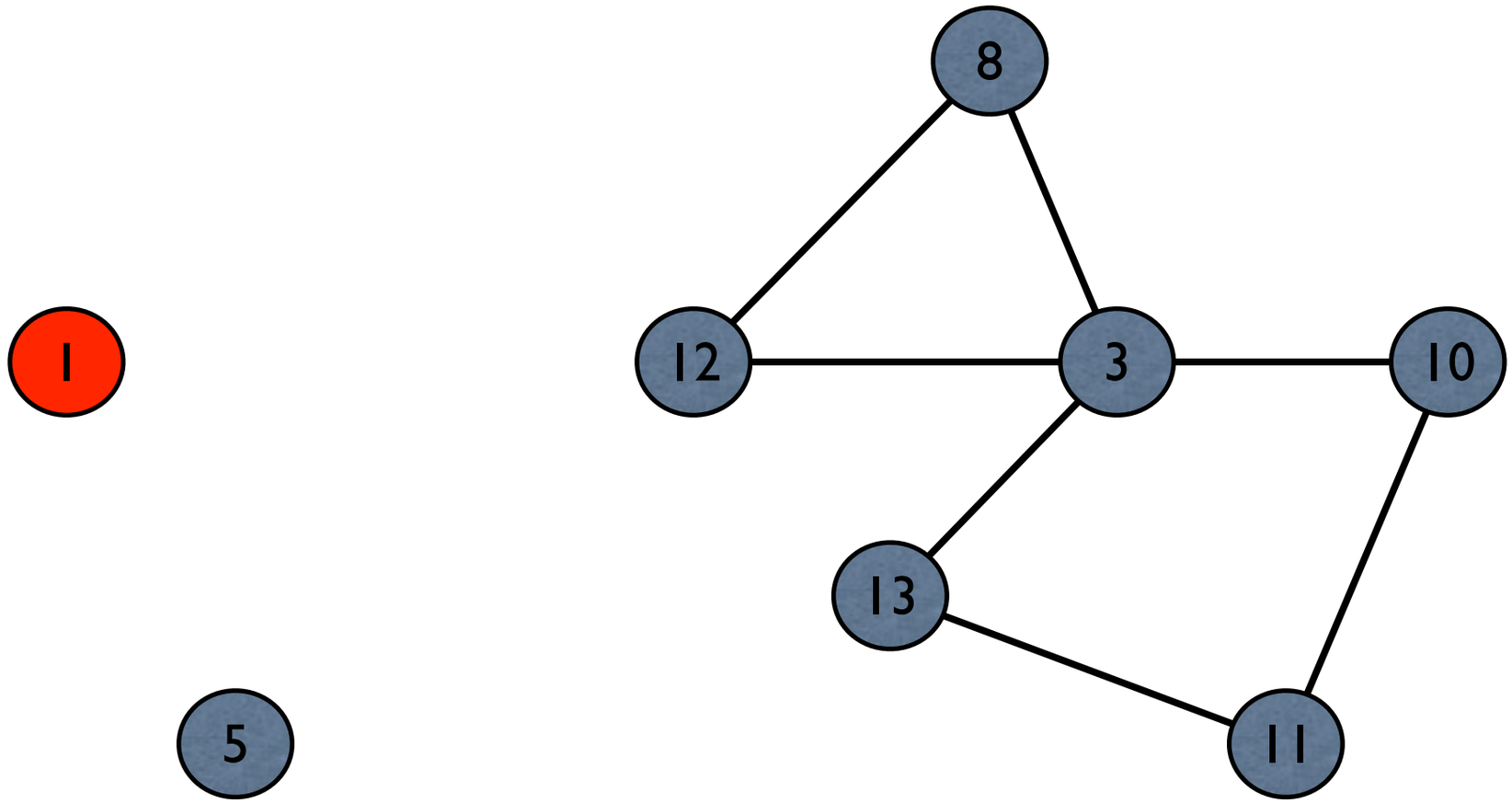

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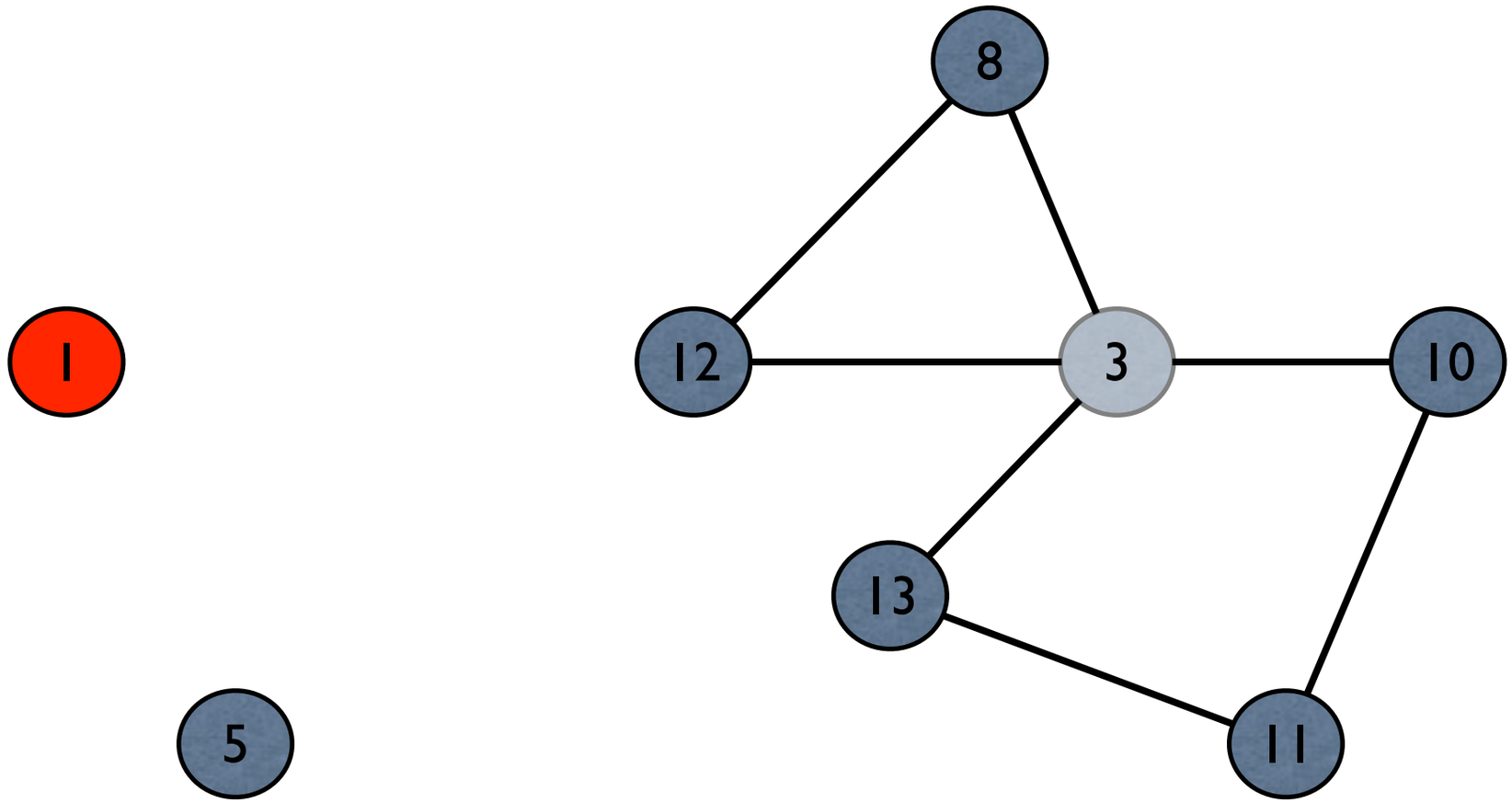

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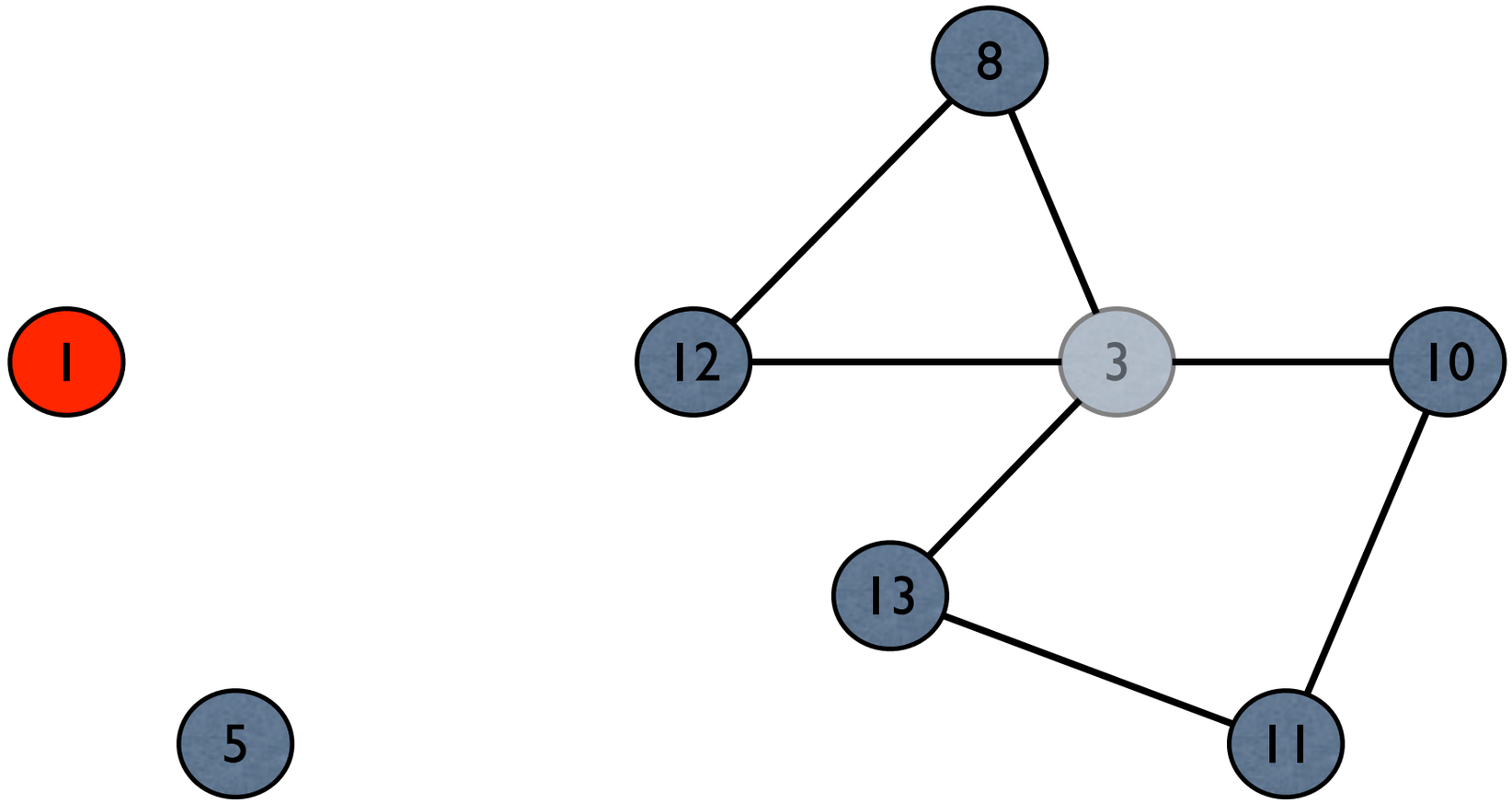

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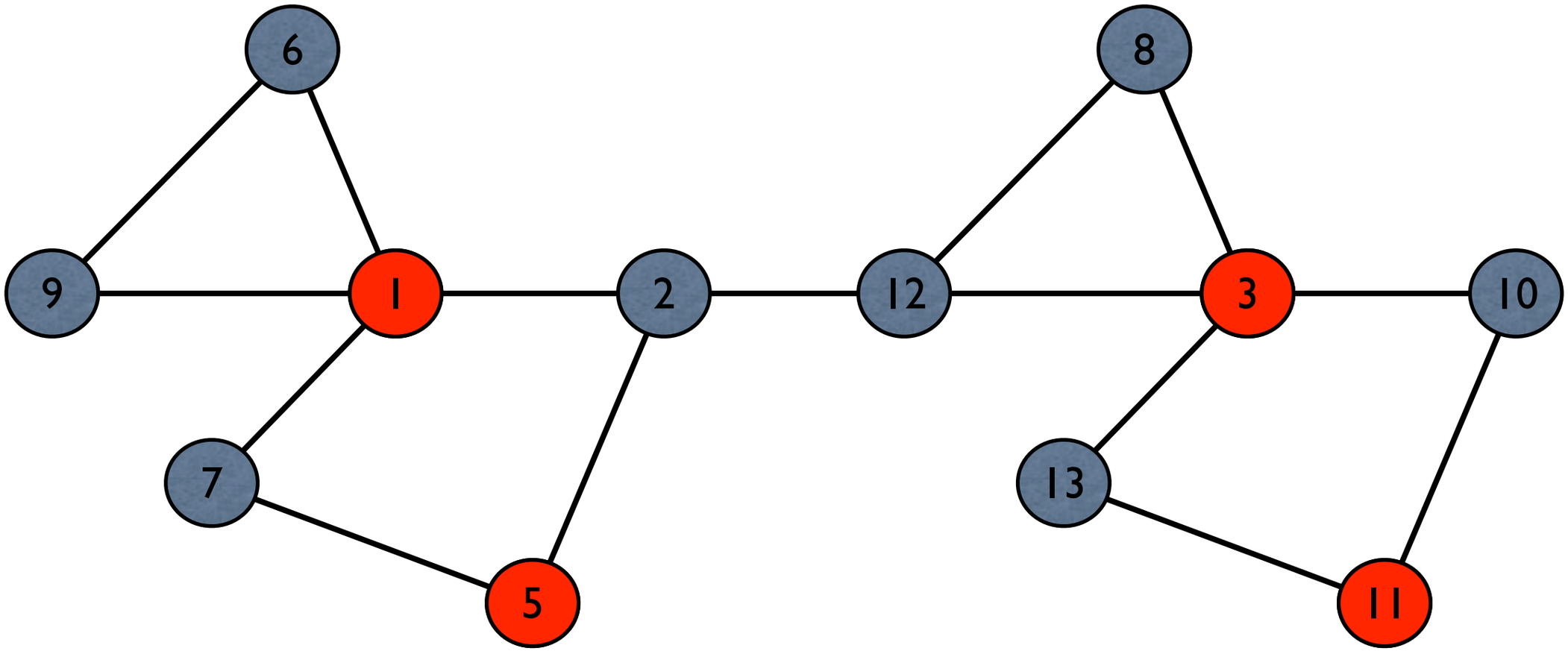
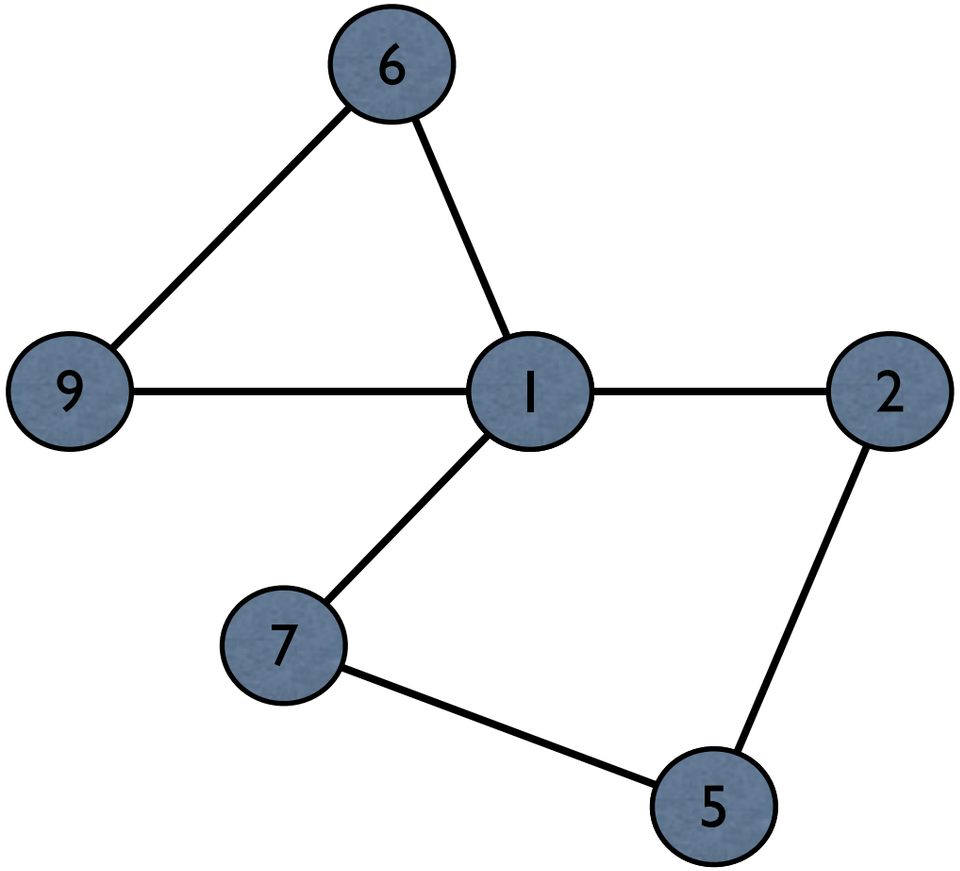


Distributed MIS



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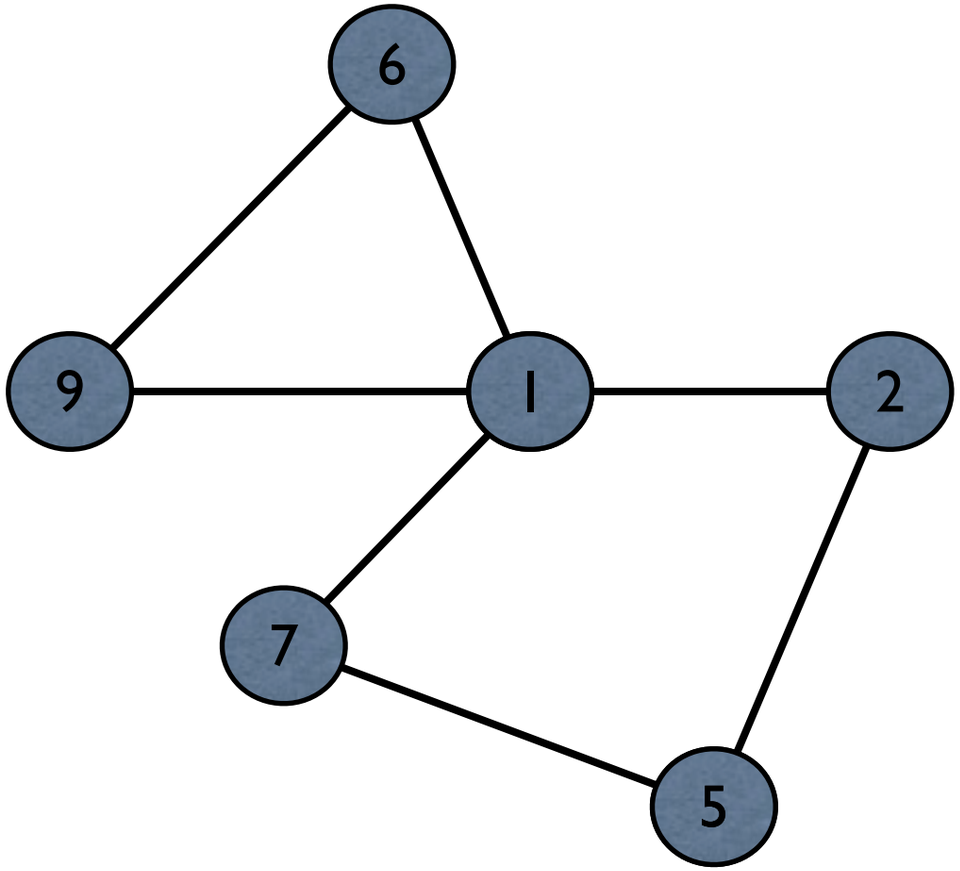

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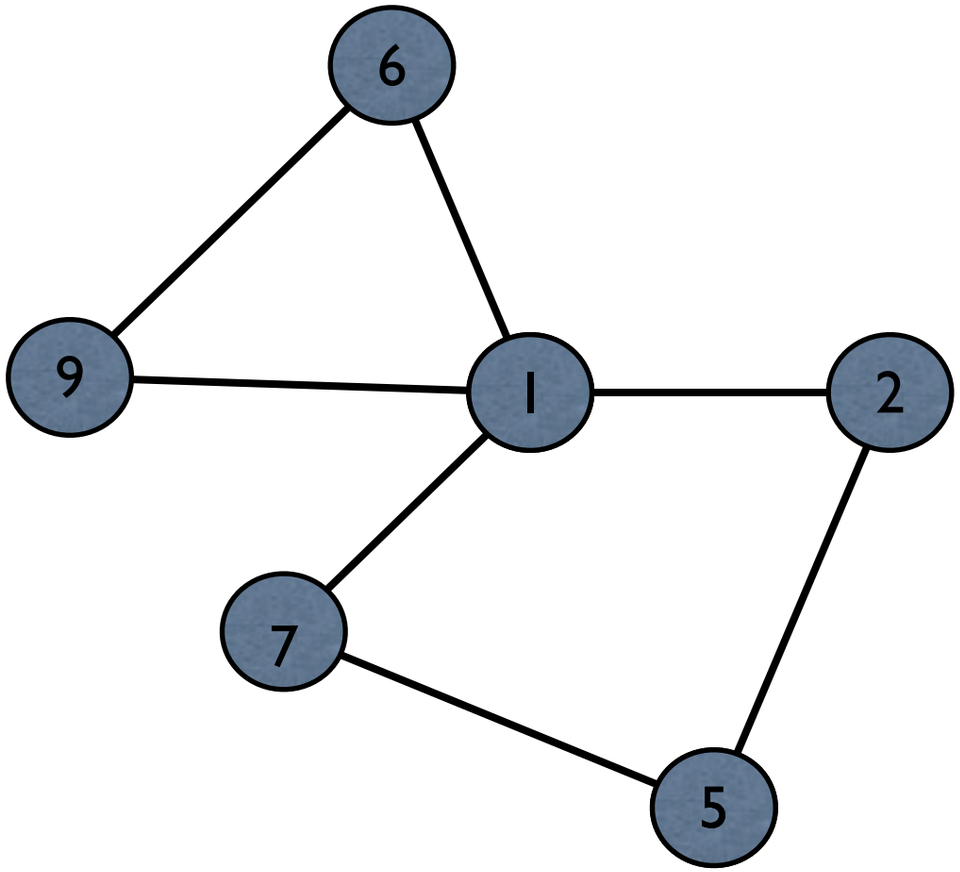
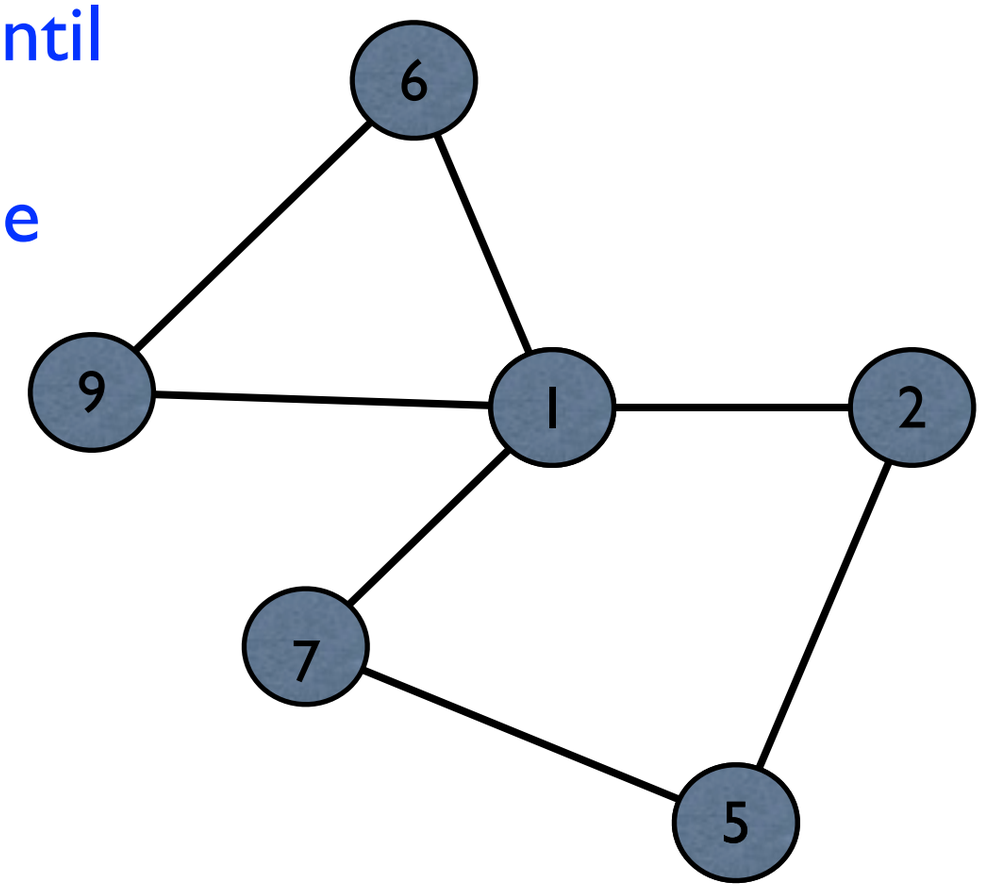


Distributed MIS

- Each node waits until neighbors with greater «id» decide



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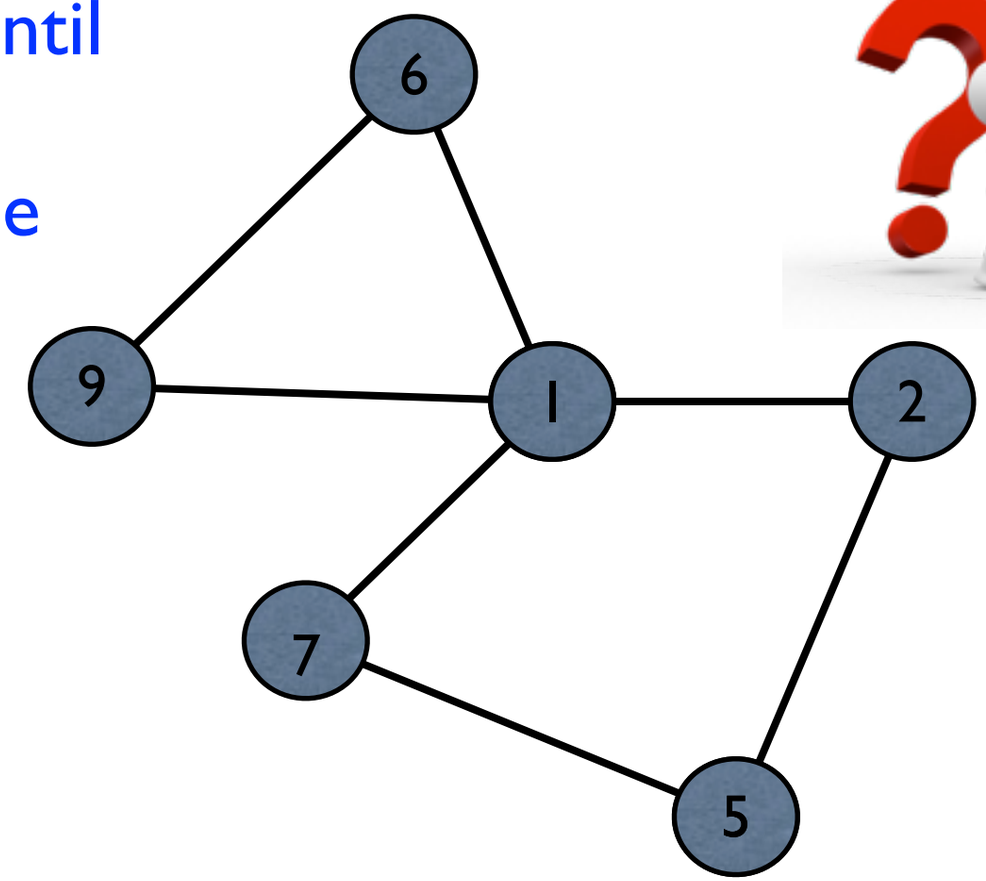

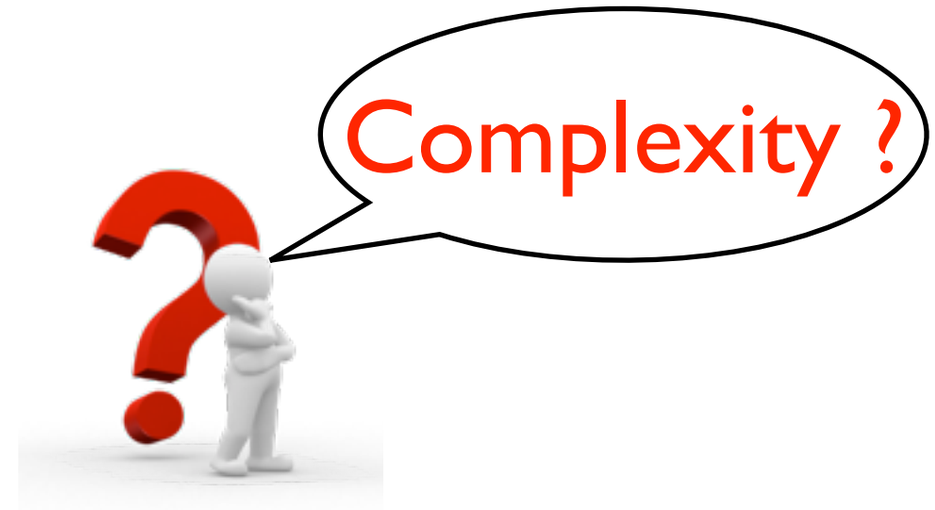

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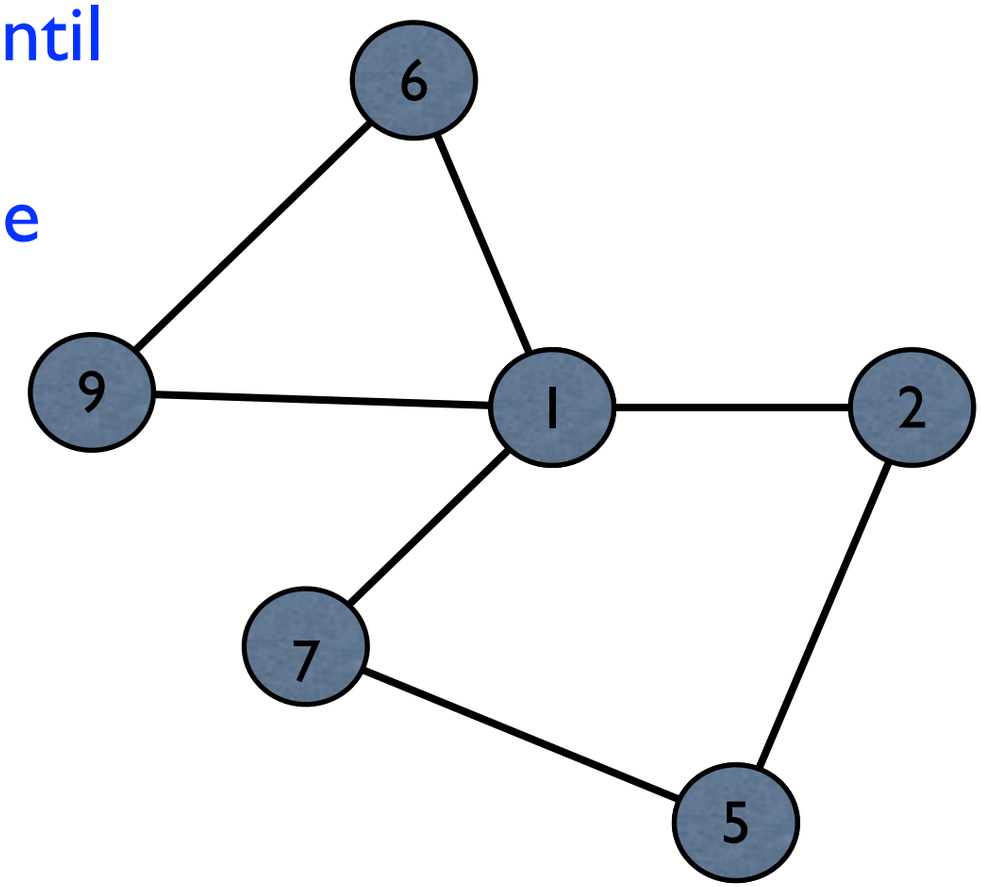

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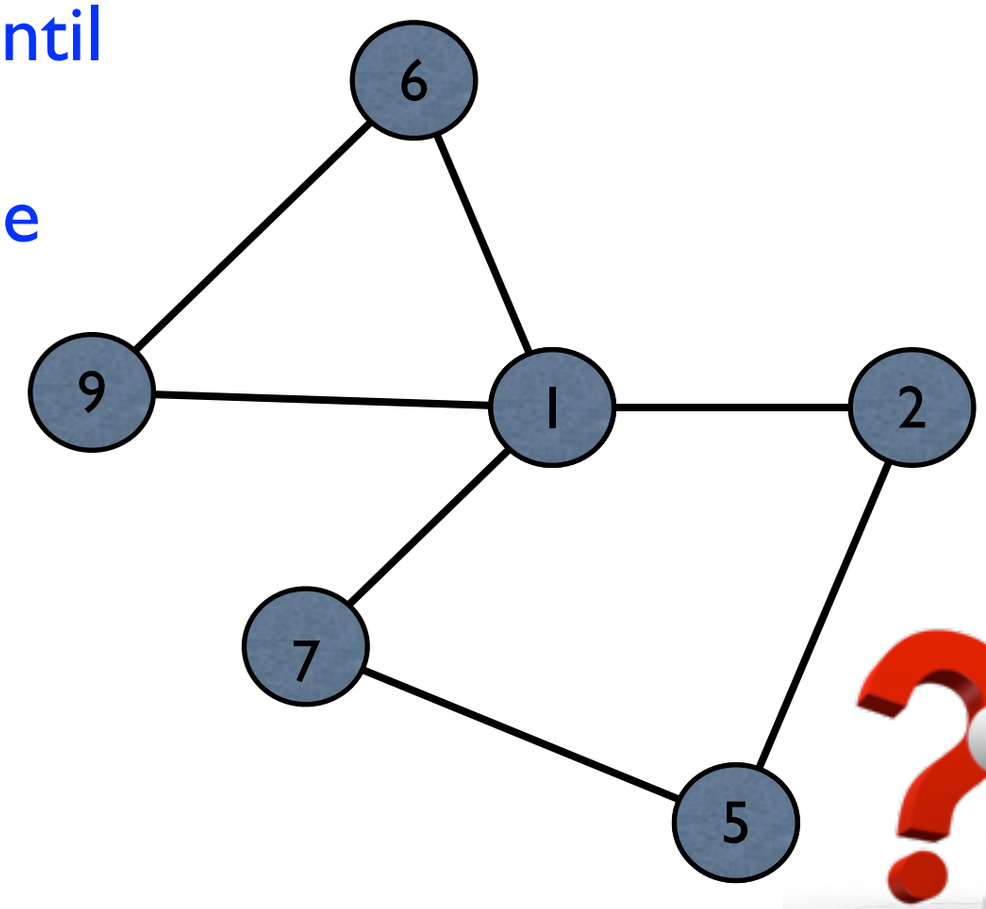

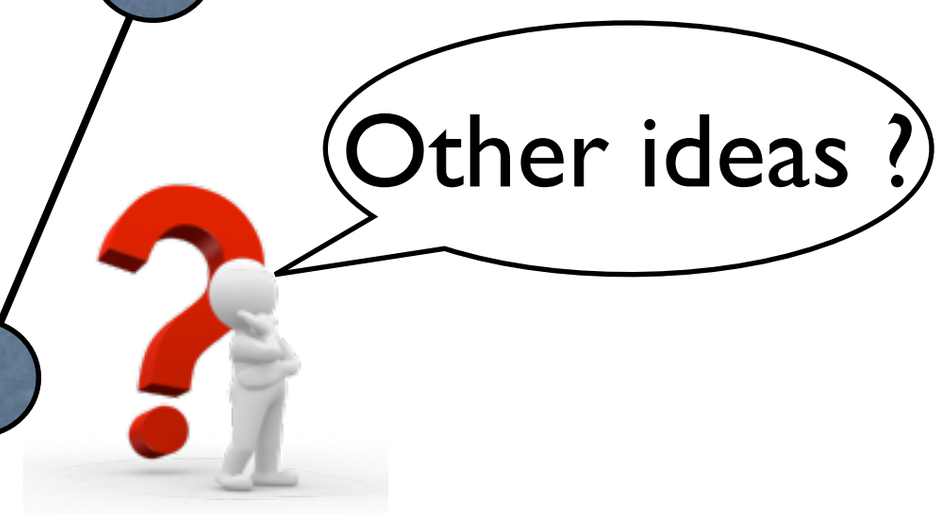

Dominating Sets

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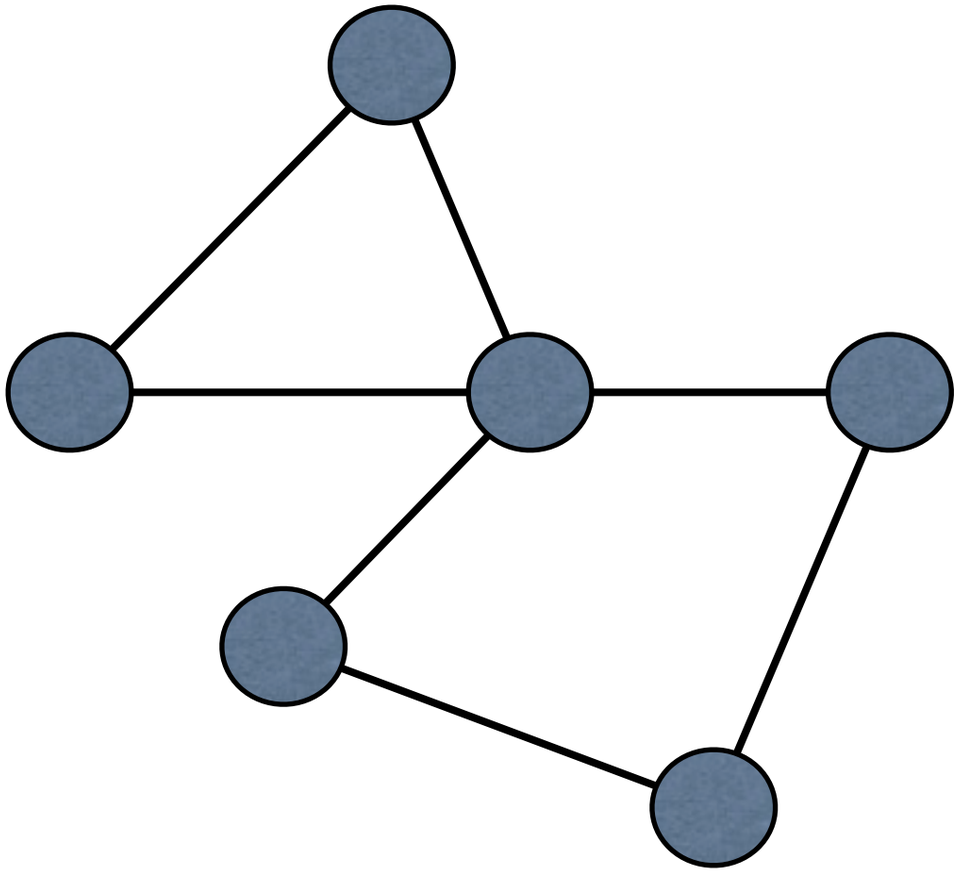

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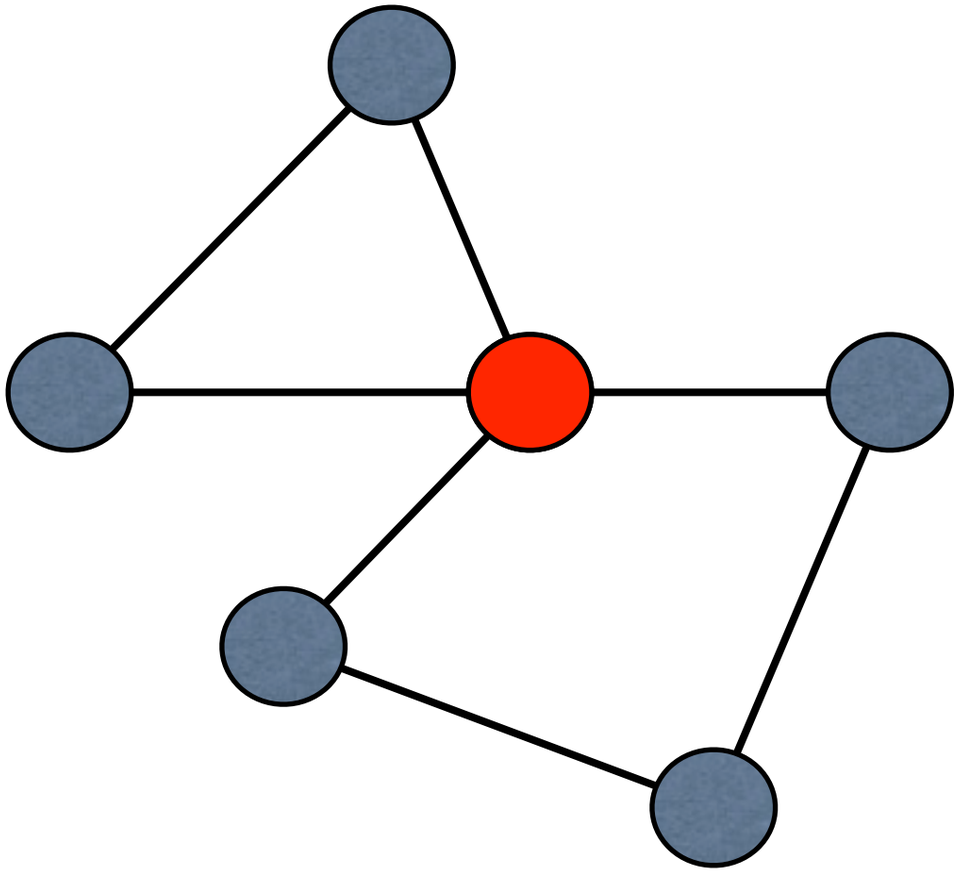

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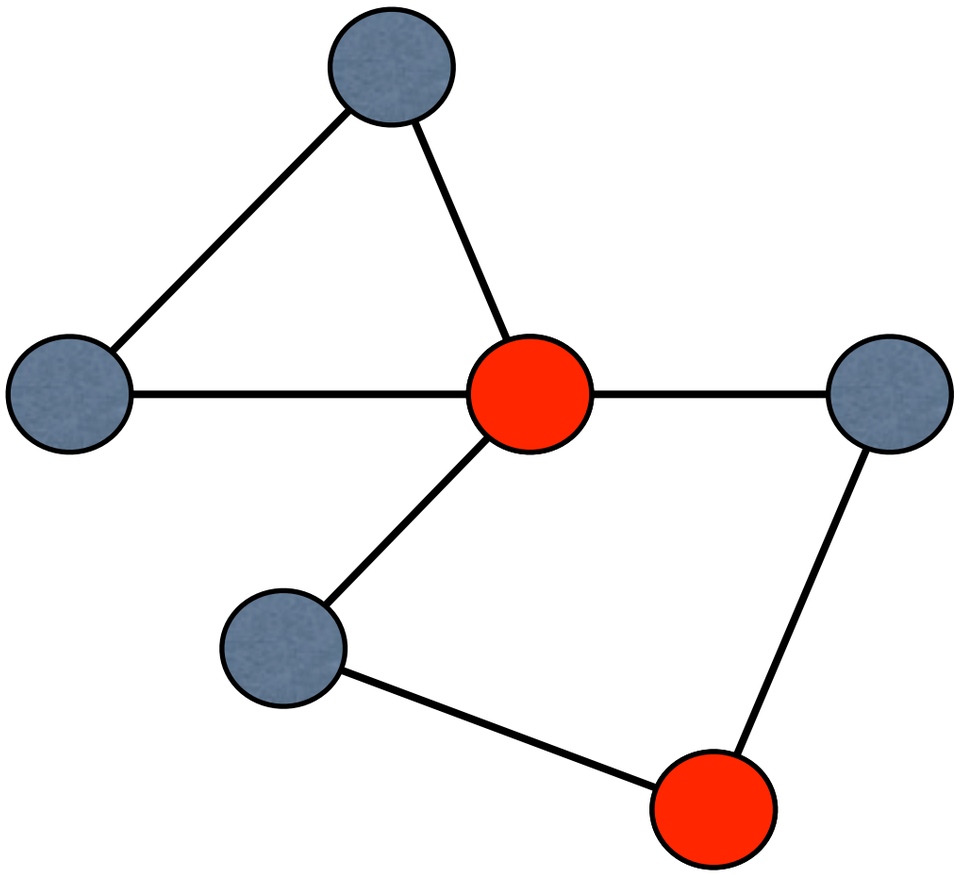

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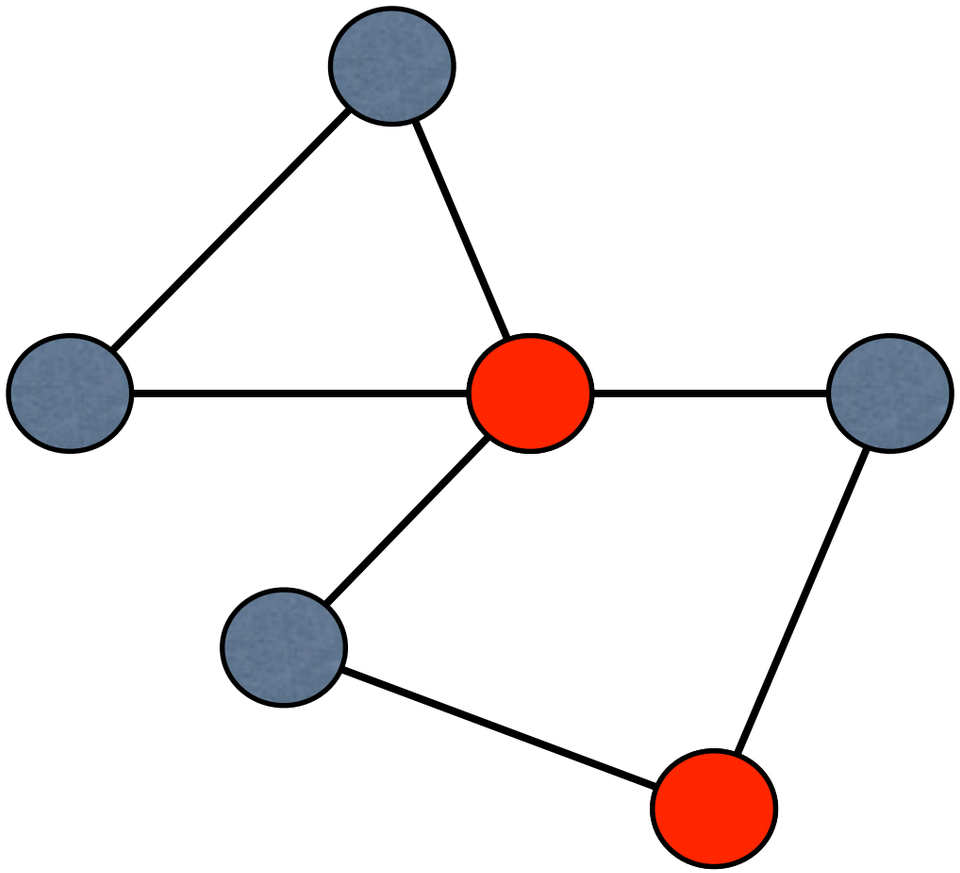
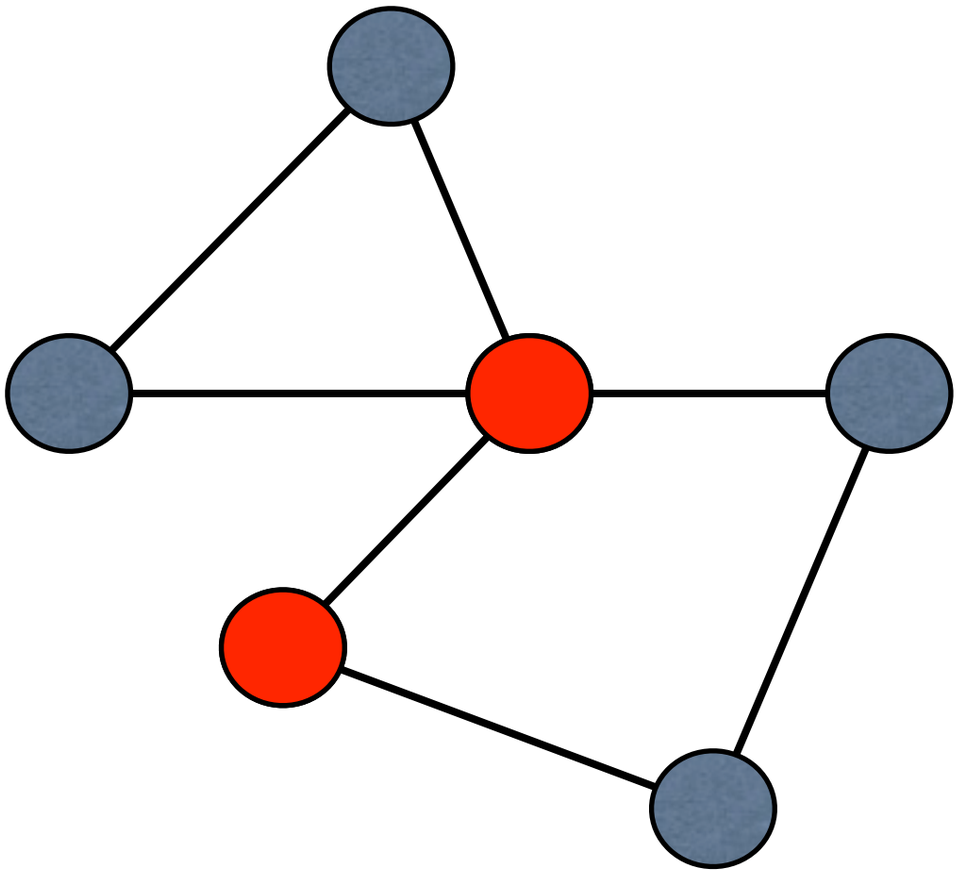
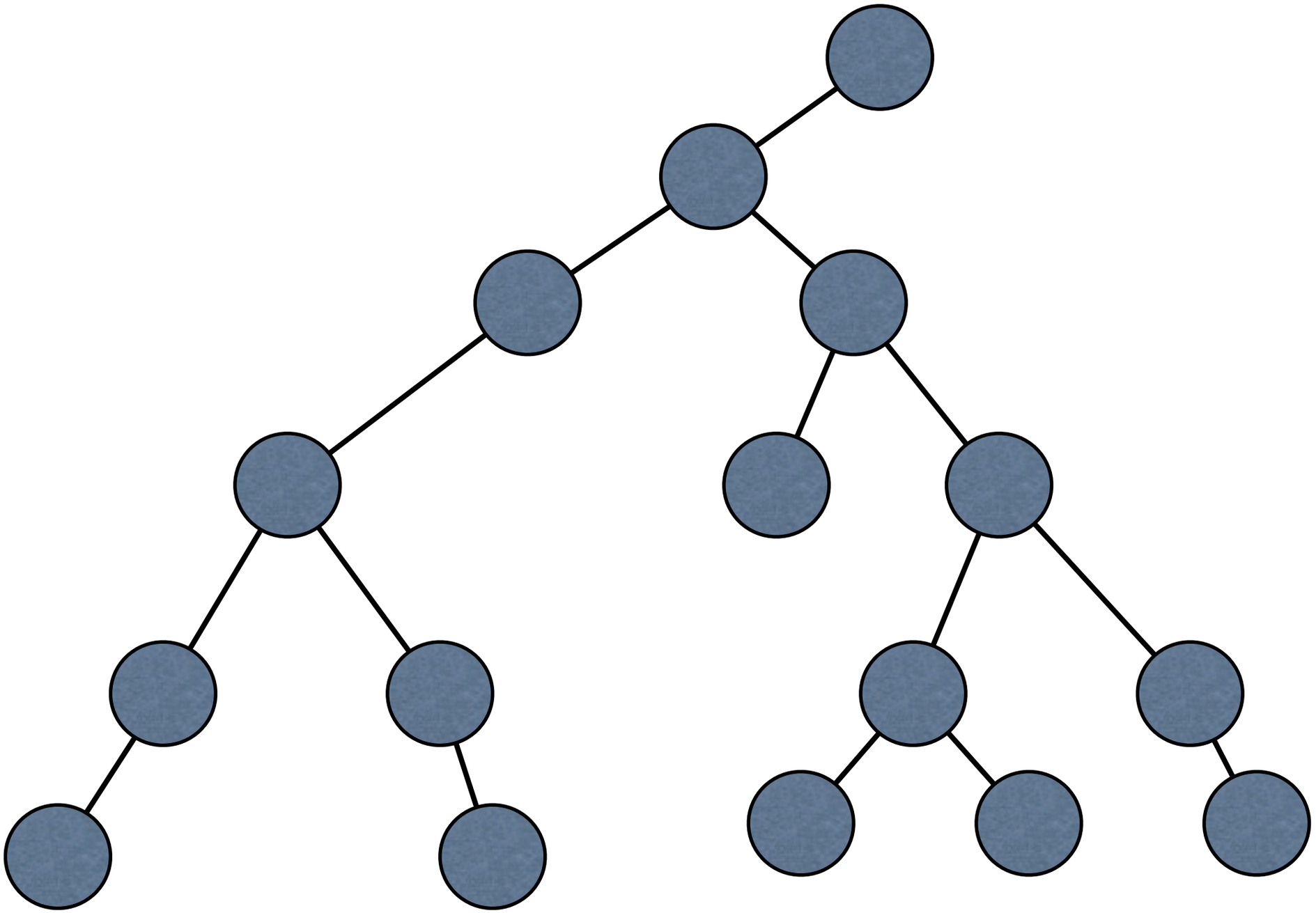




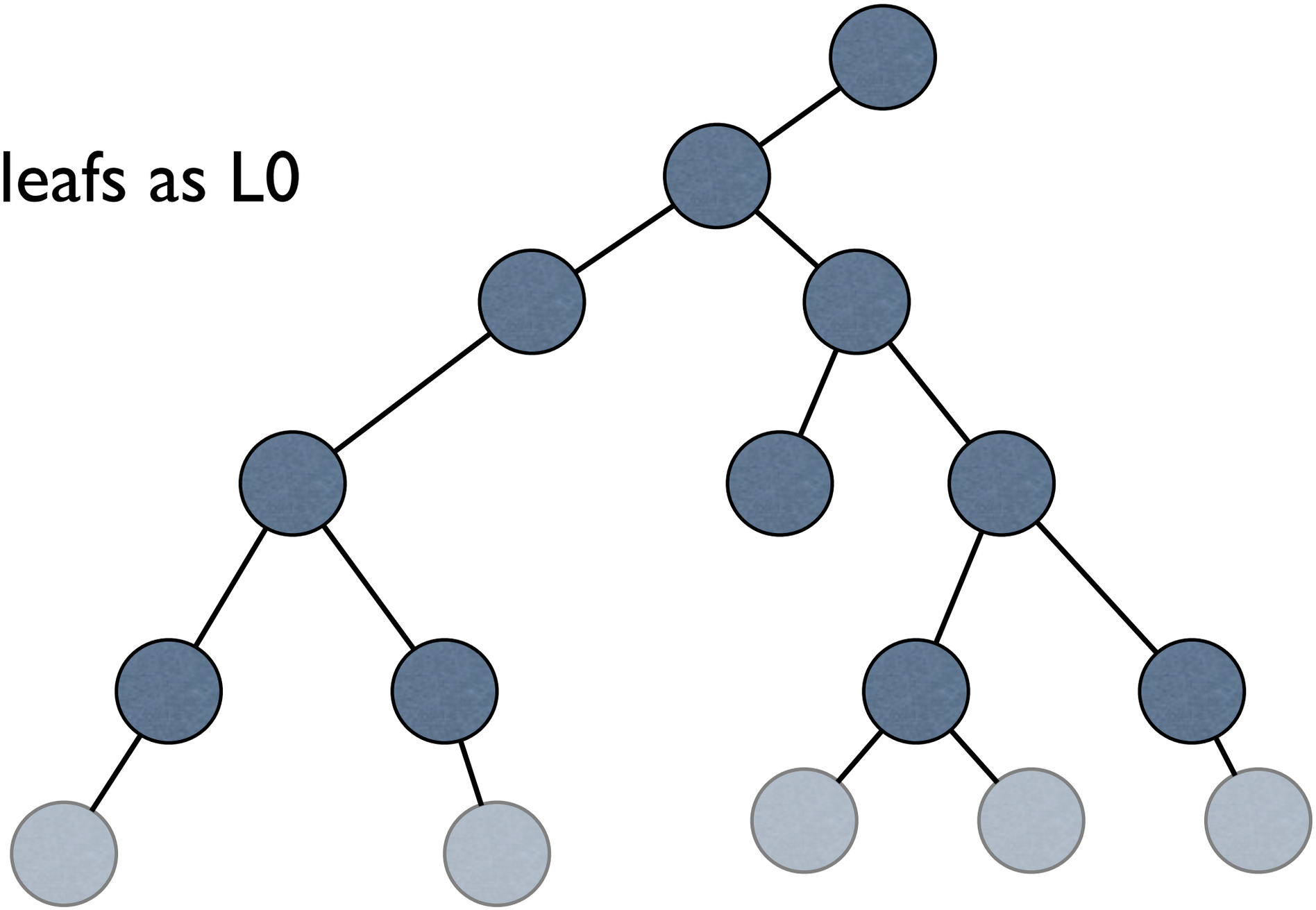
DS Implementation

DS on Trees



DS on Trees

Mark leafs as L0



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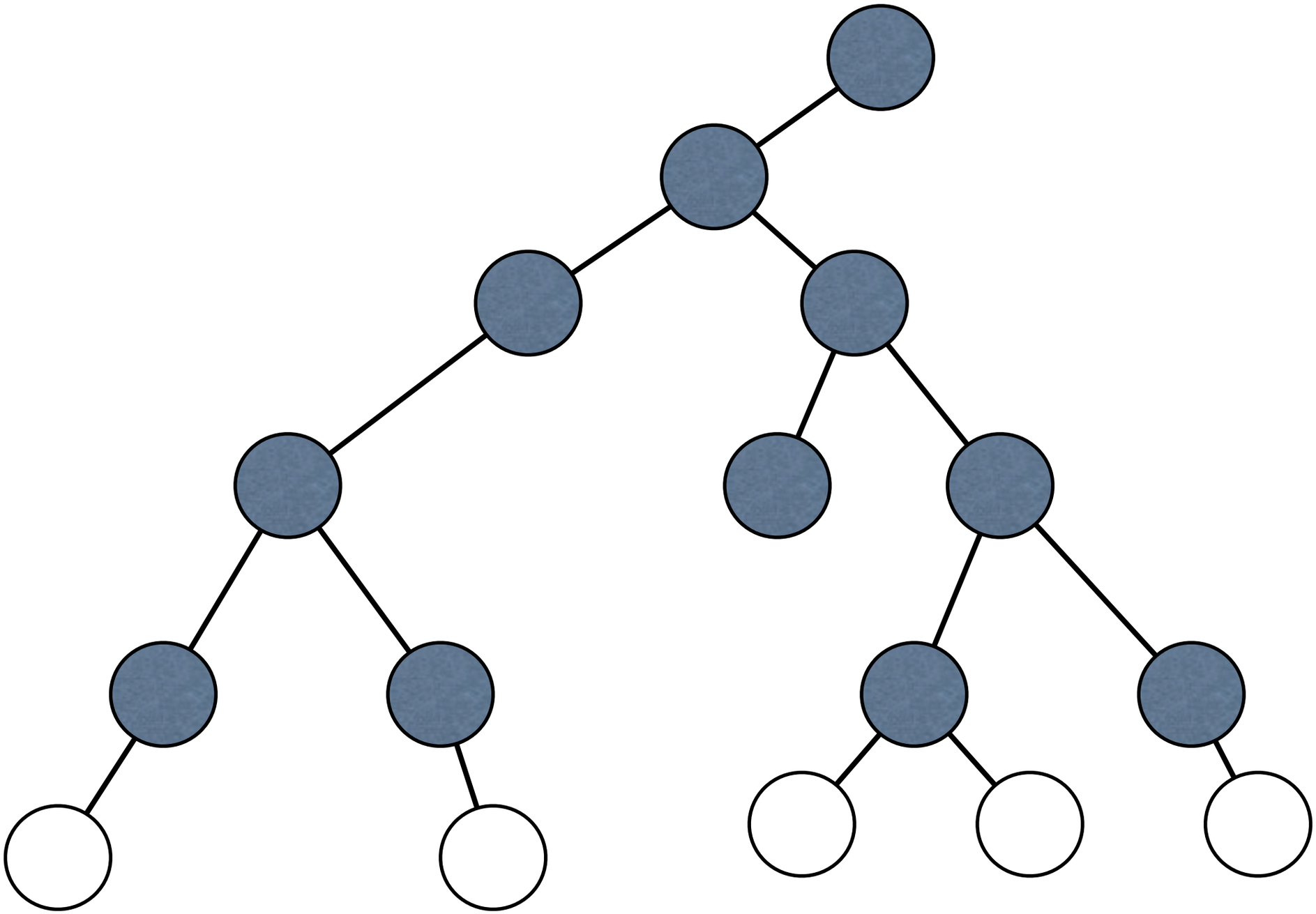
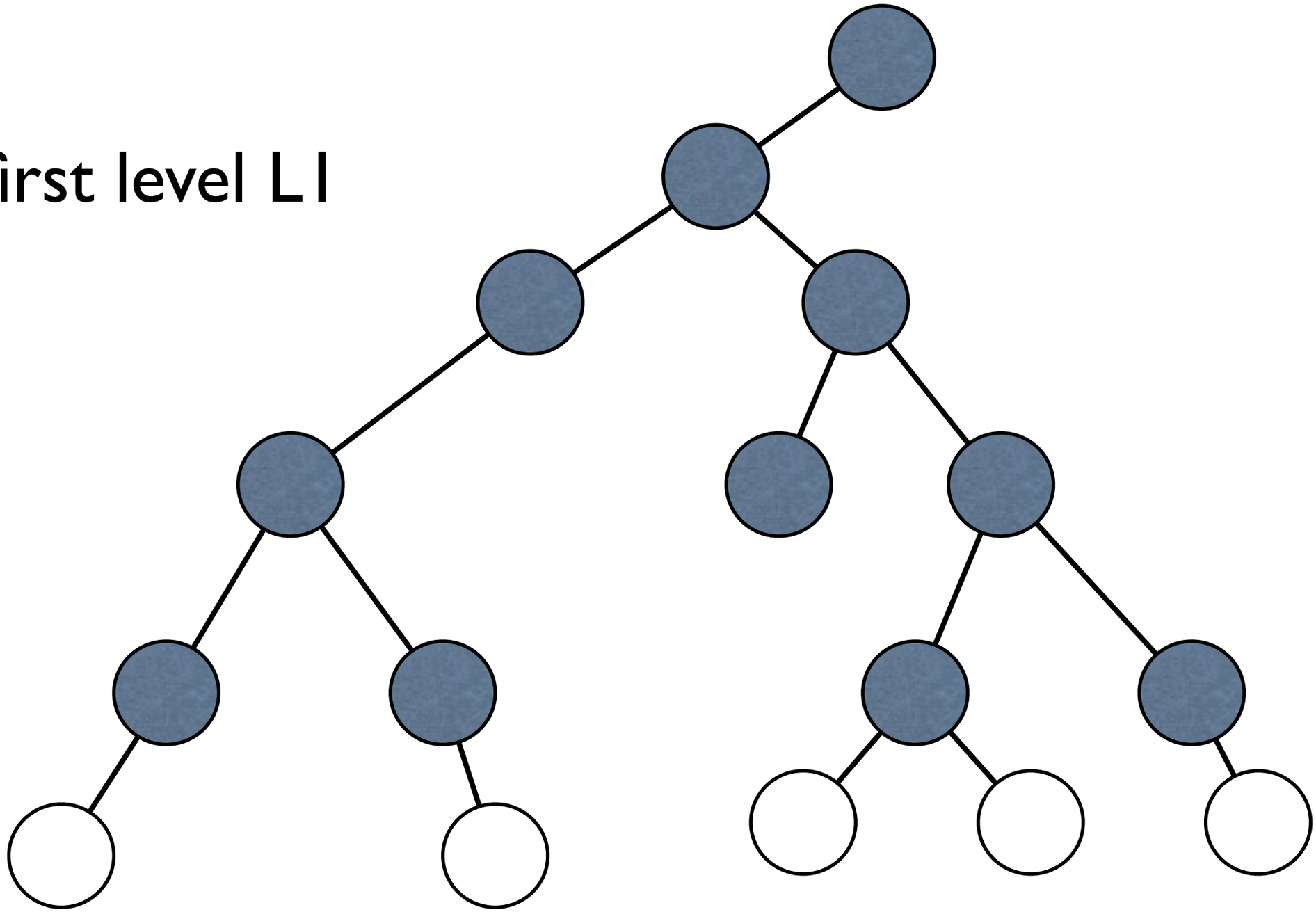


DS on Trees

Mark first level L1



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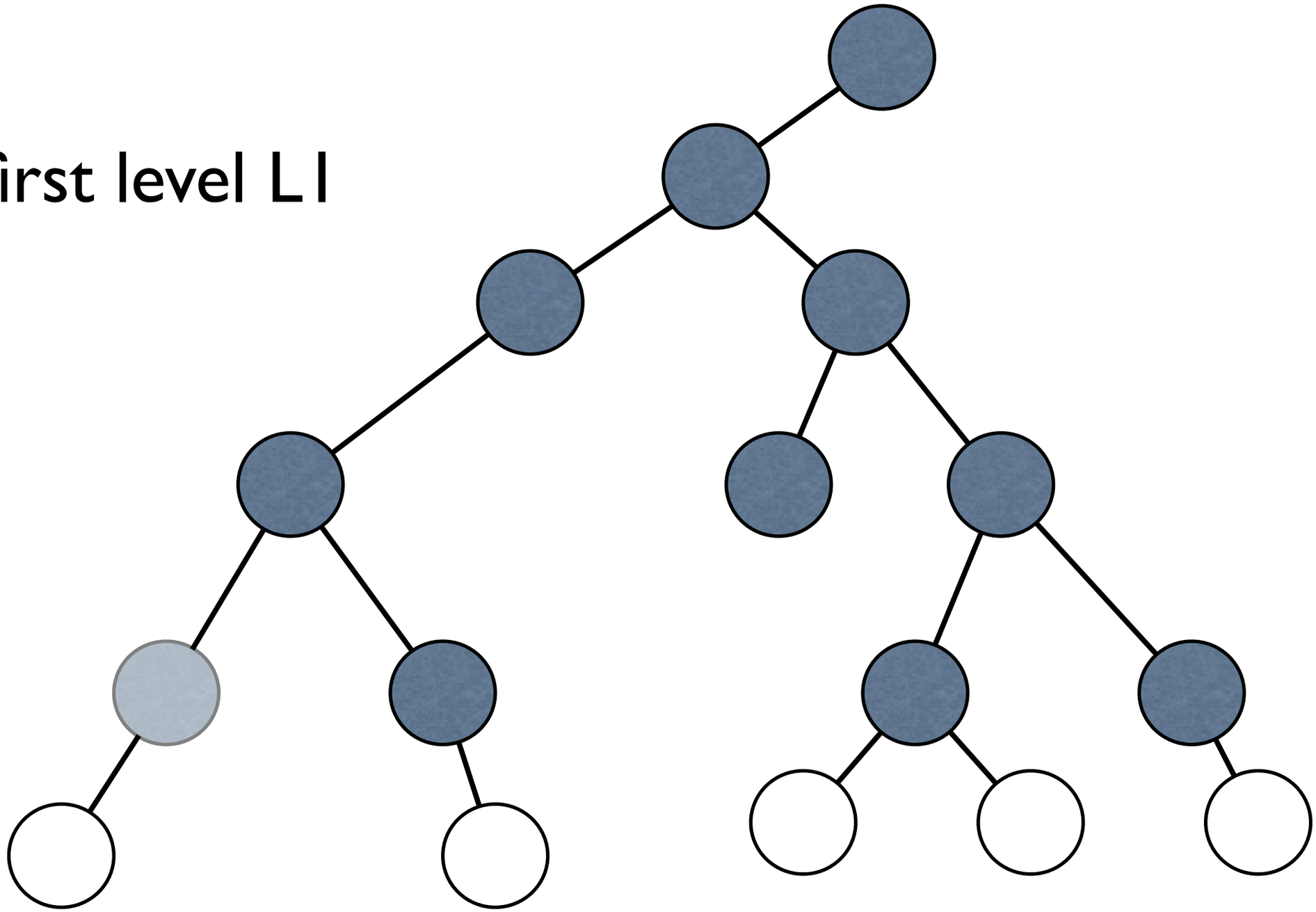

DS on Trees

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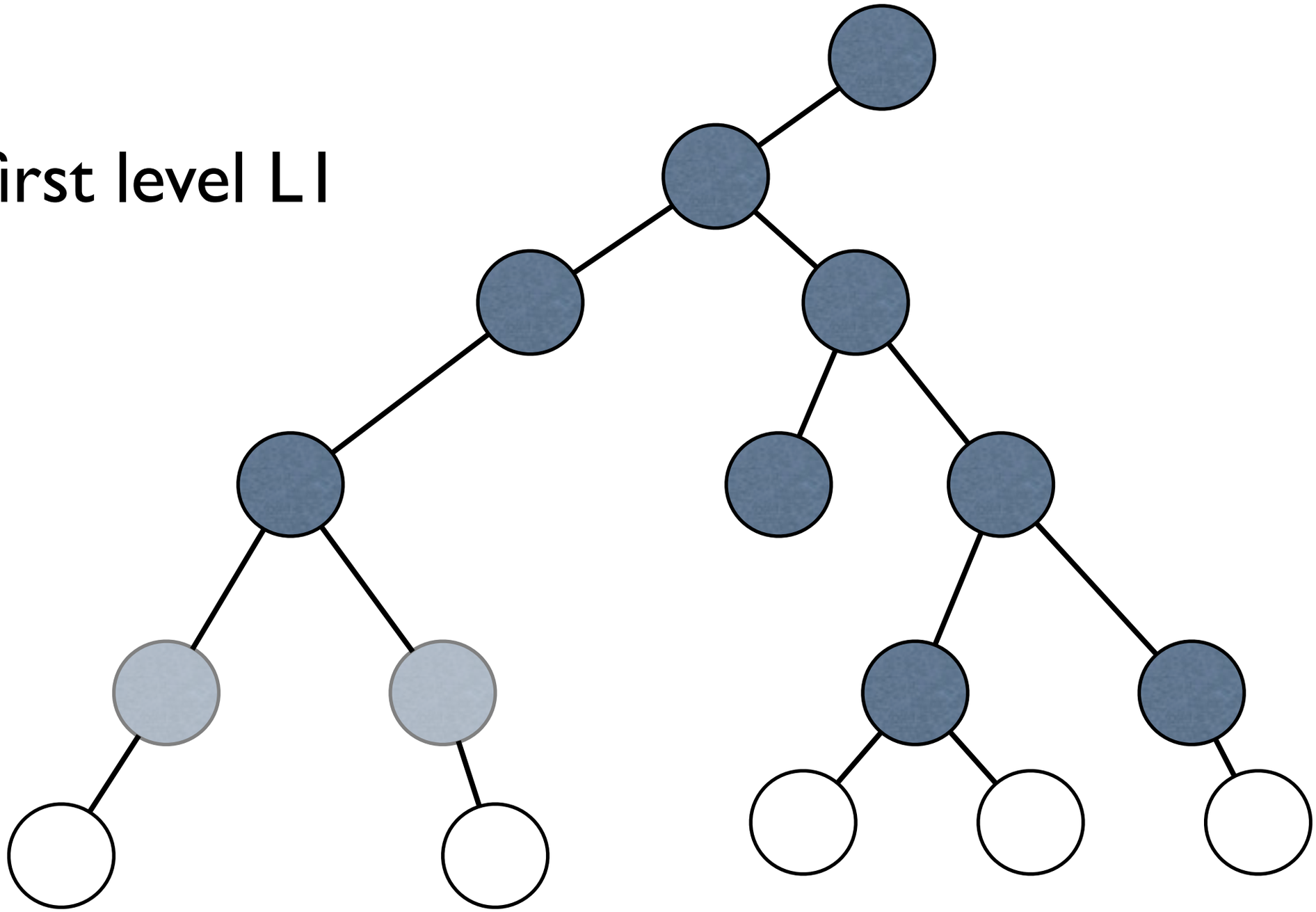

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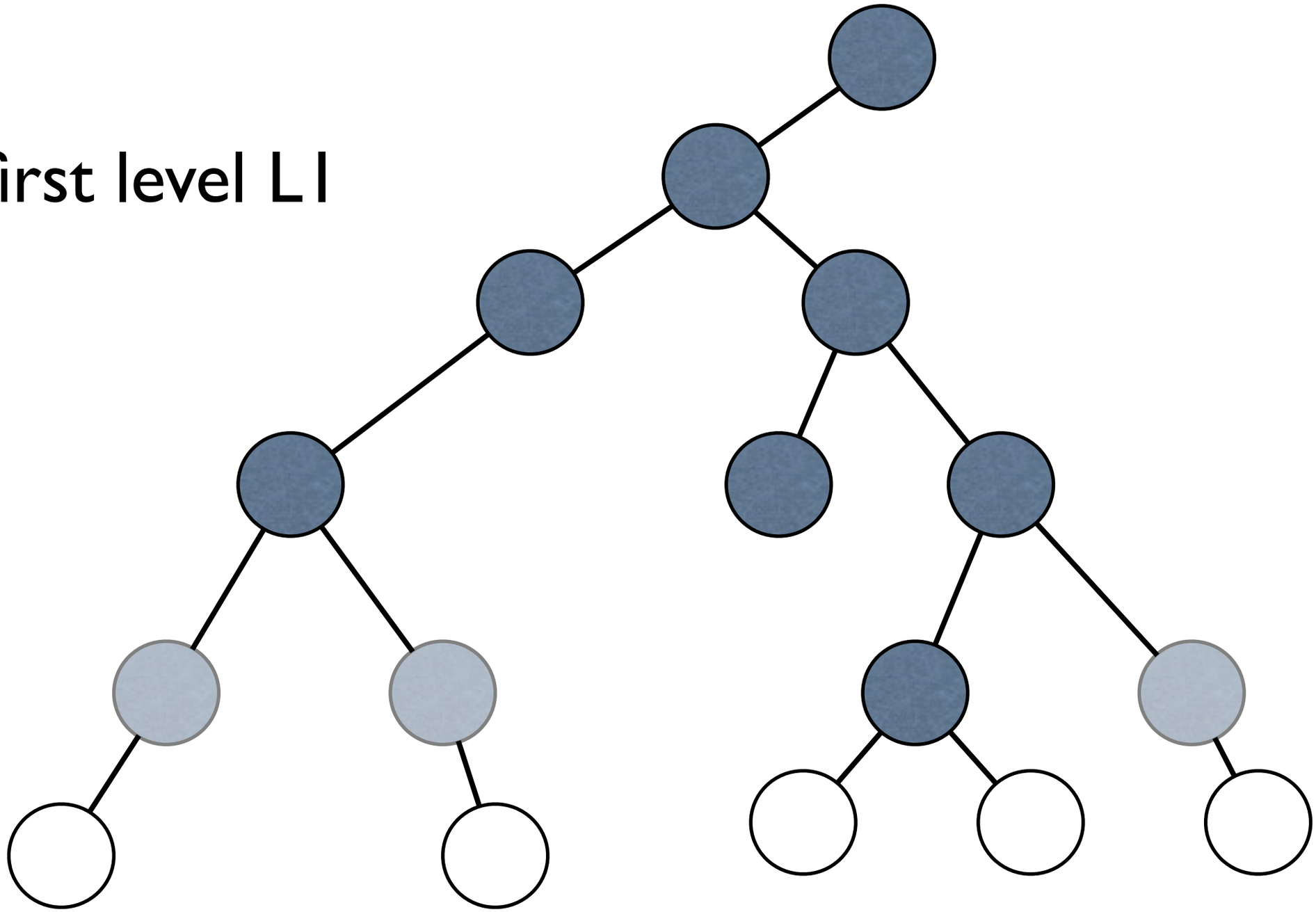

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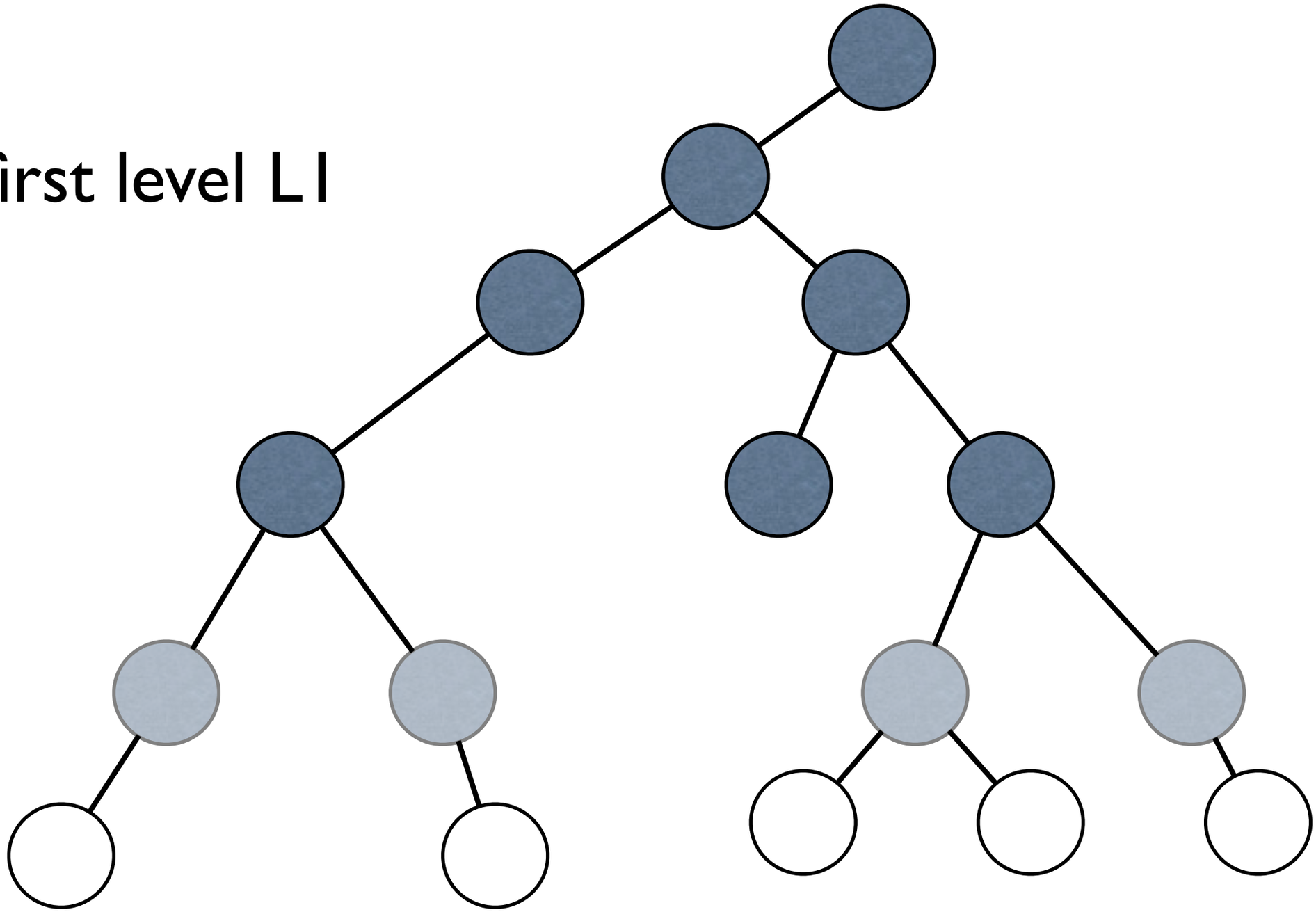

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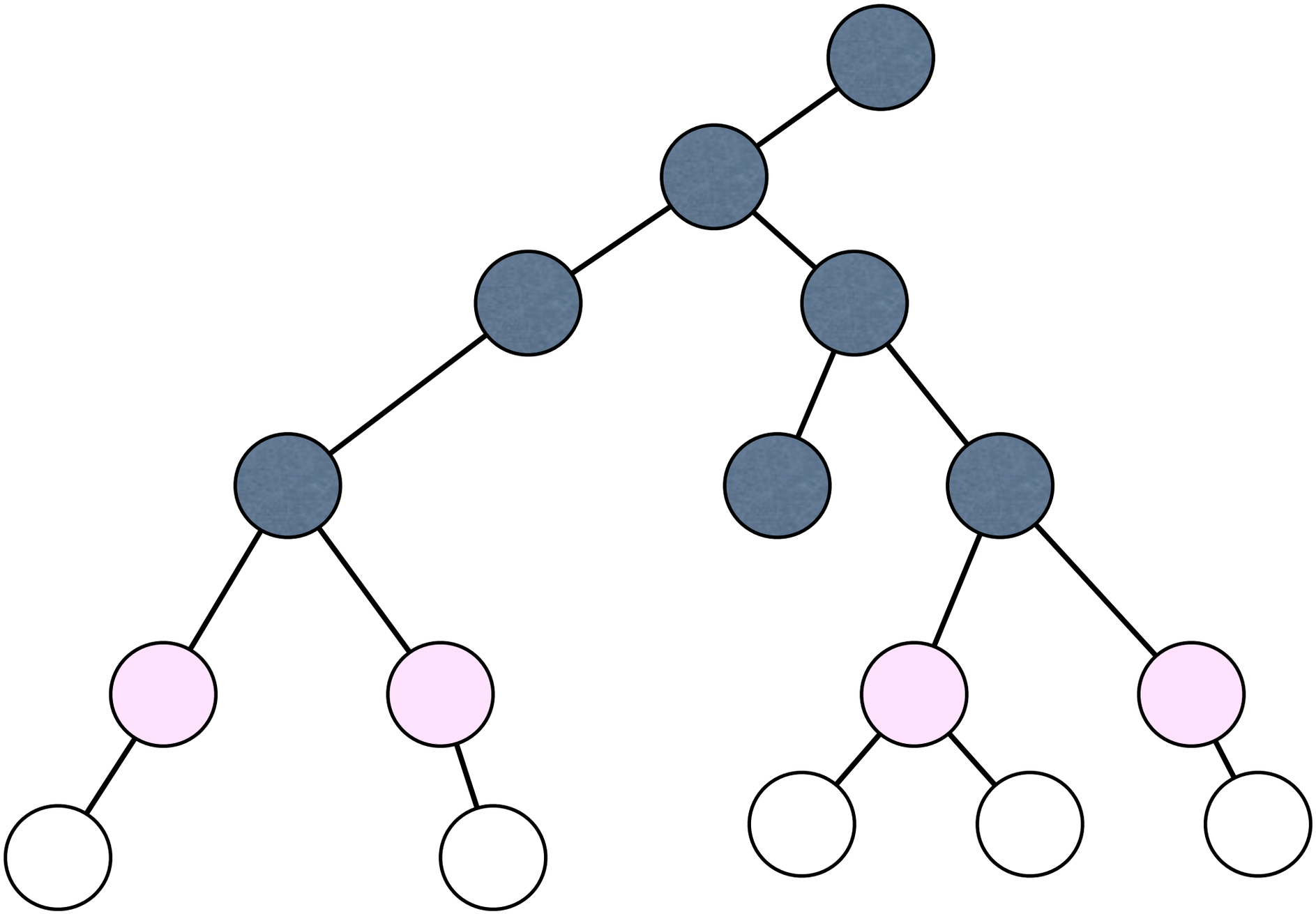
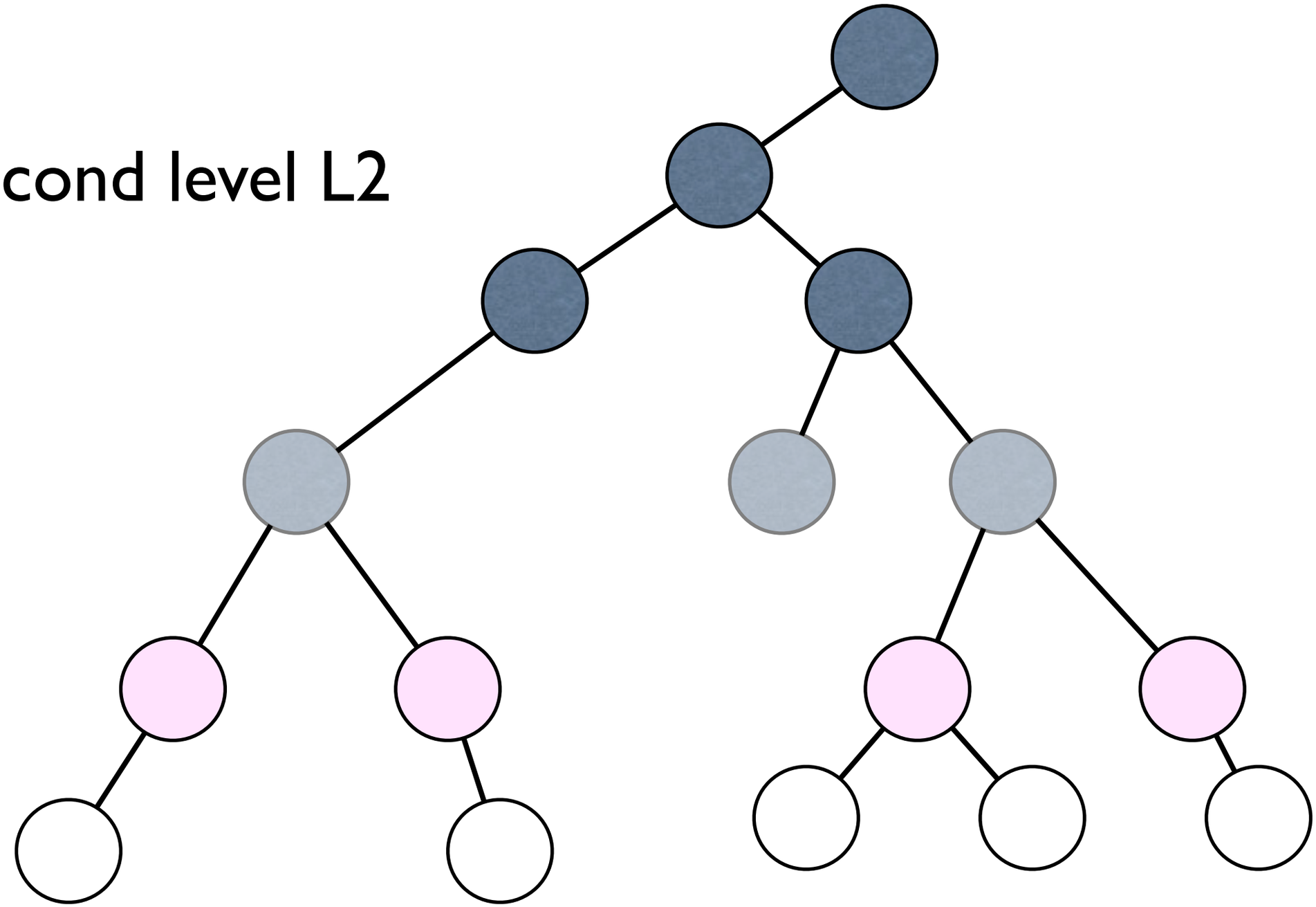
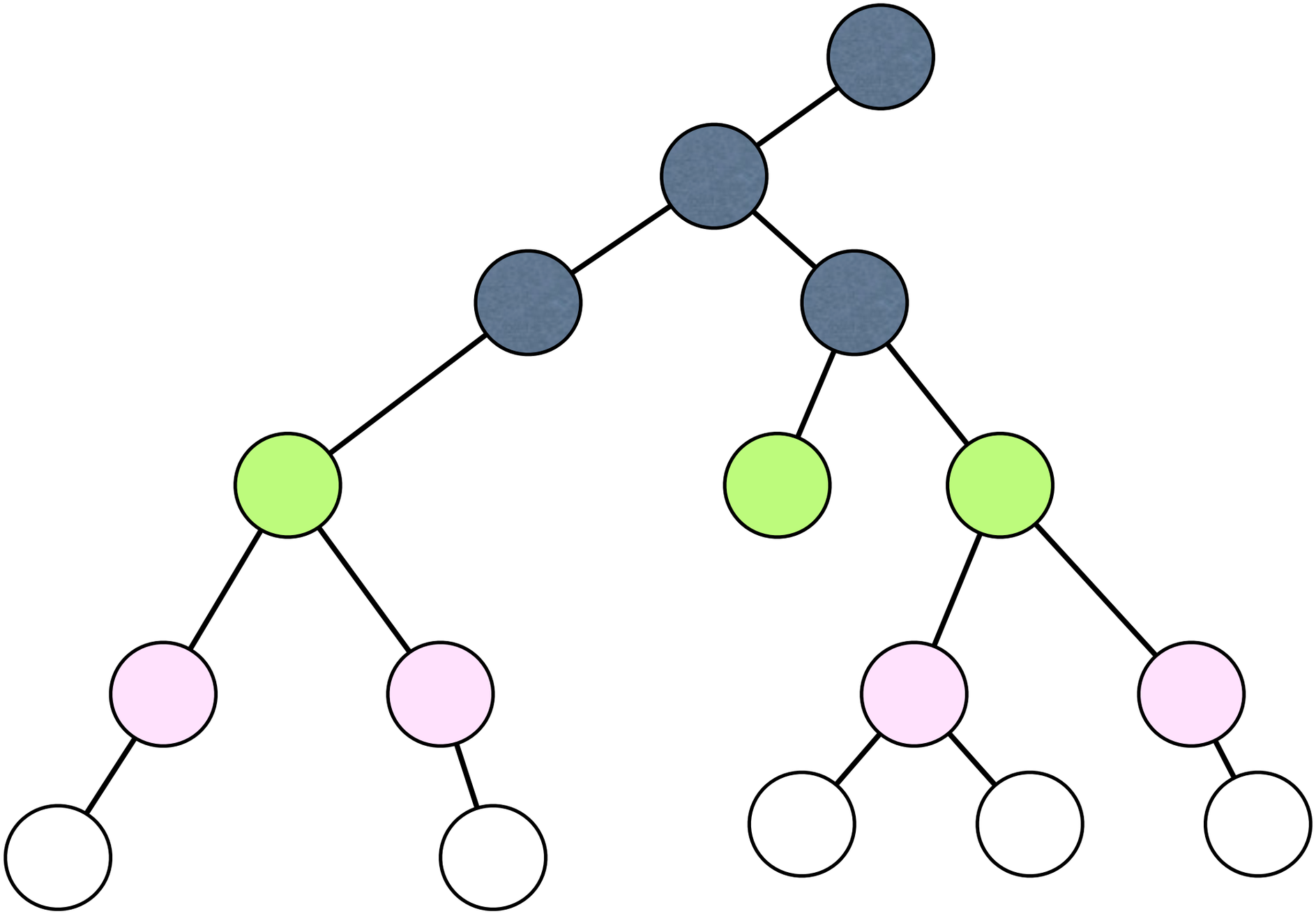


DS on Trees

Mark second level L2

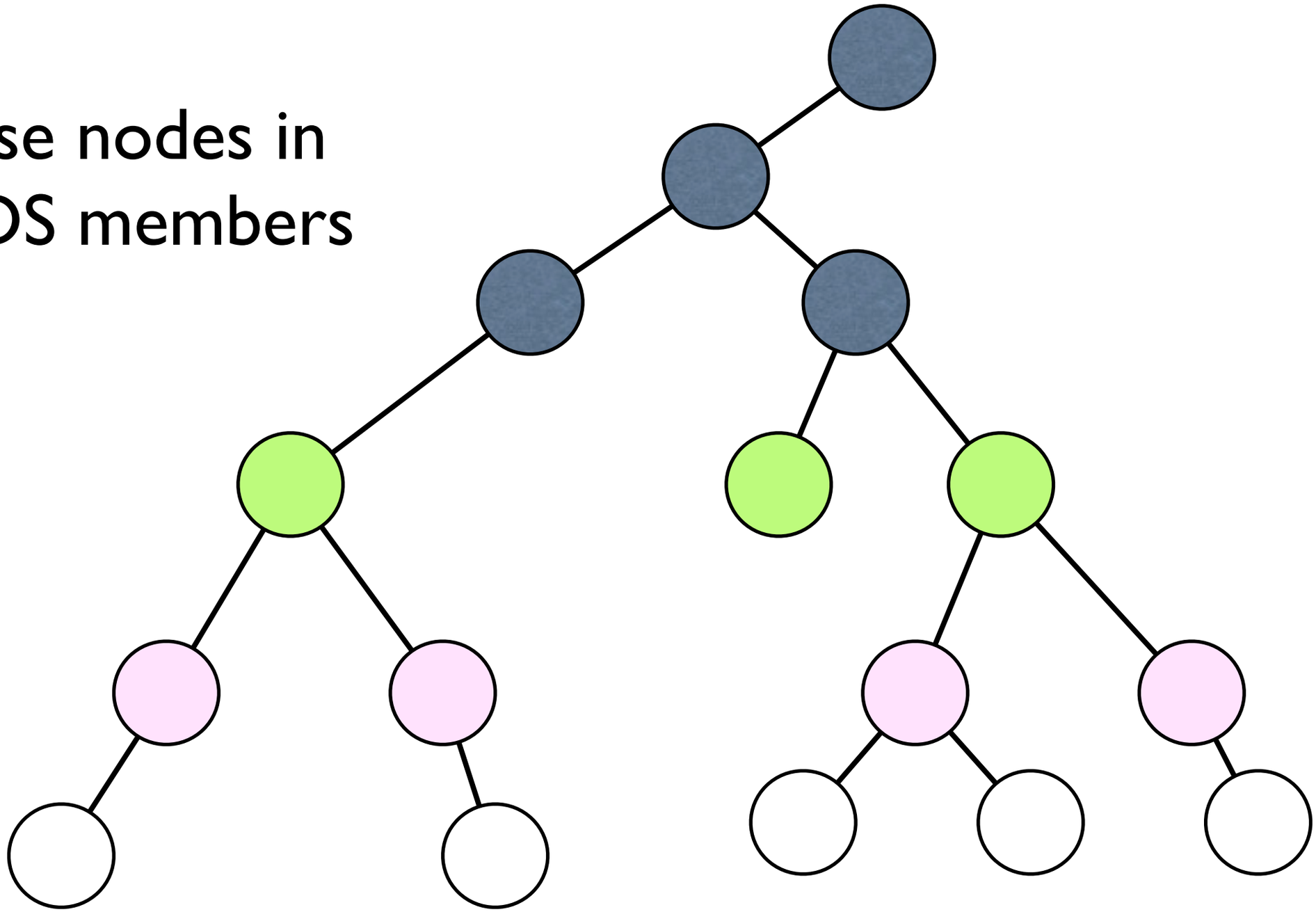


DS on Trees



DS on Trees

Choose nodes in
L1 as DS members



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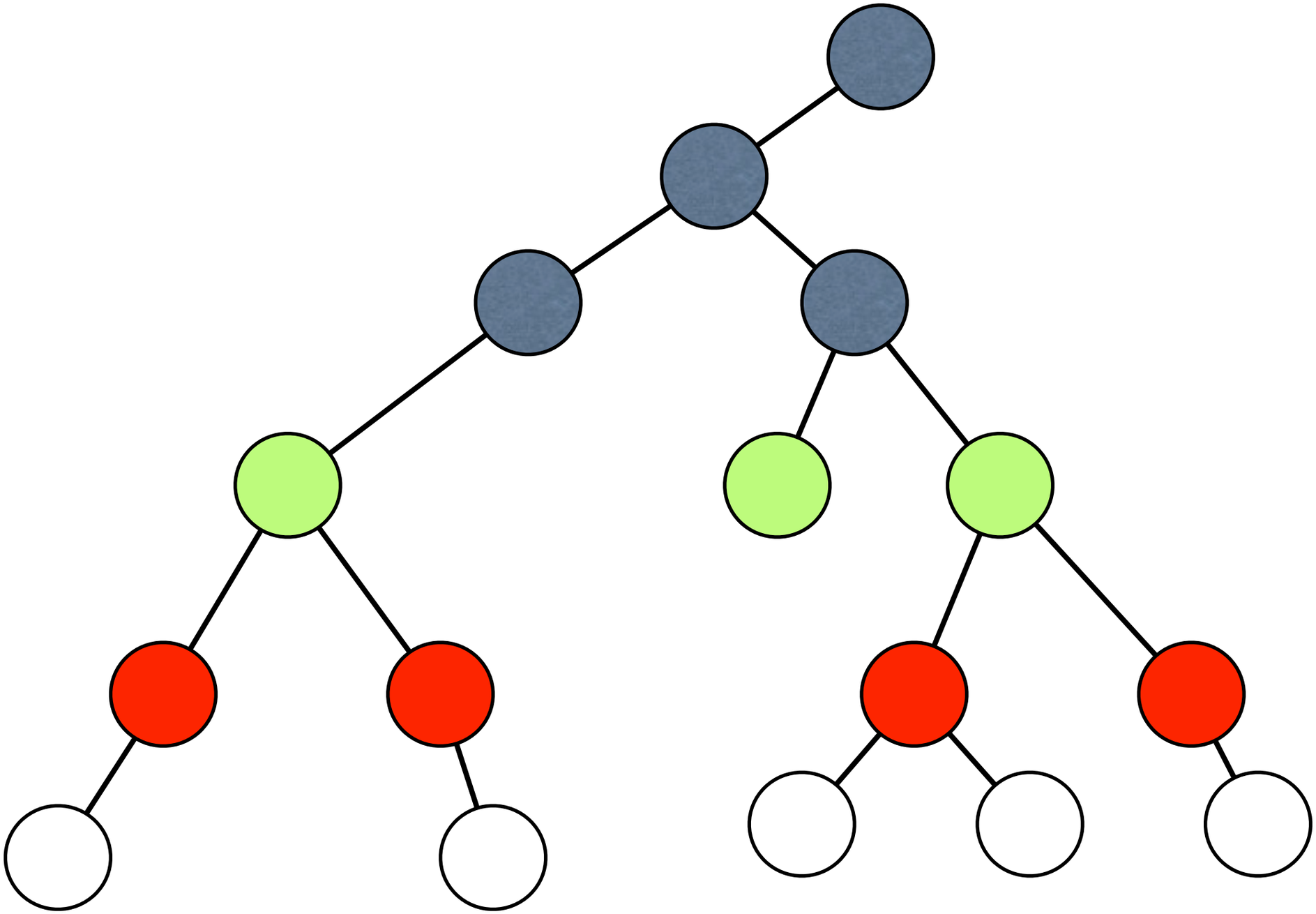
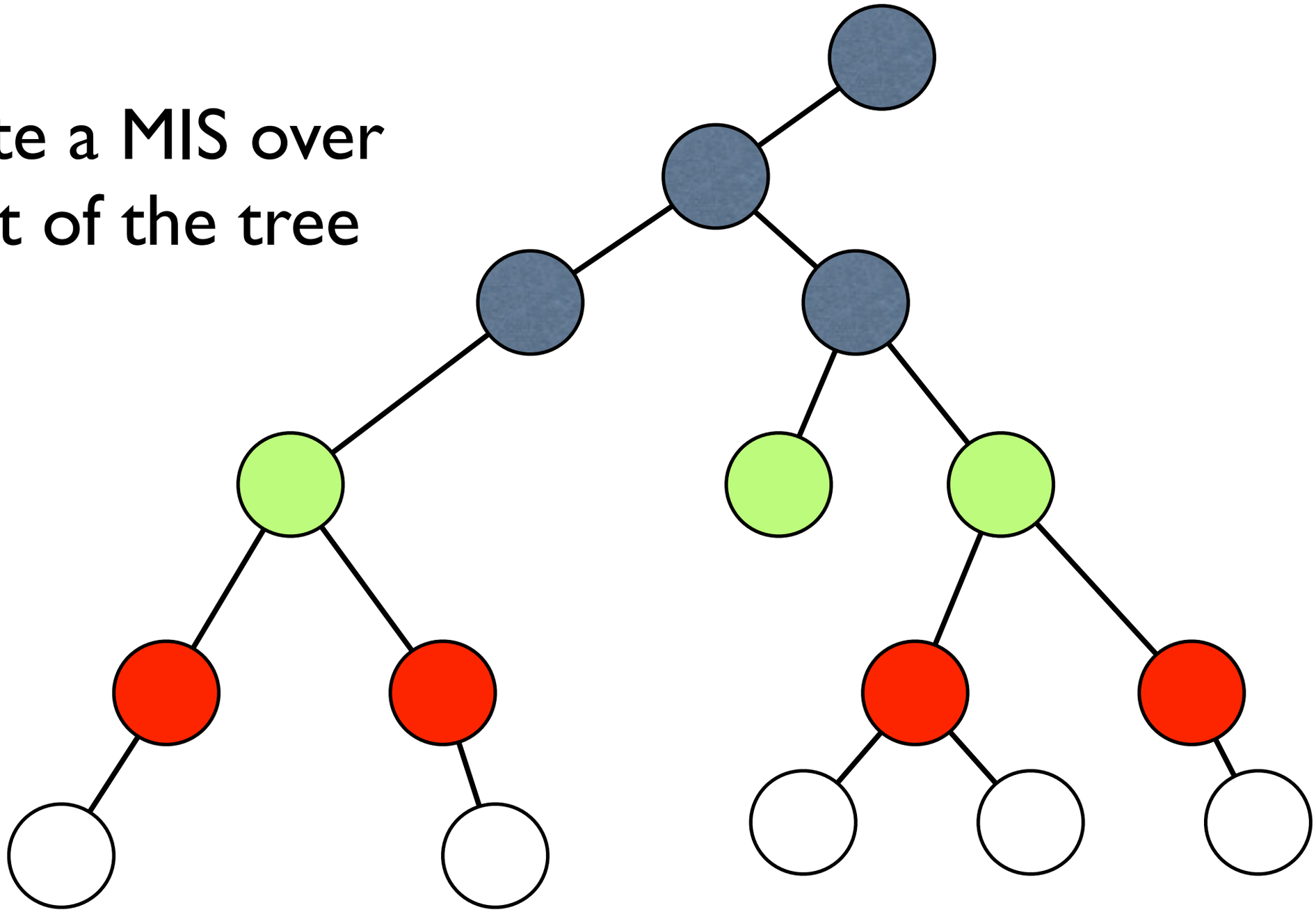


DS on Trees

Compute a MIS over the rest of the tree



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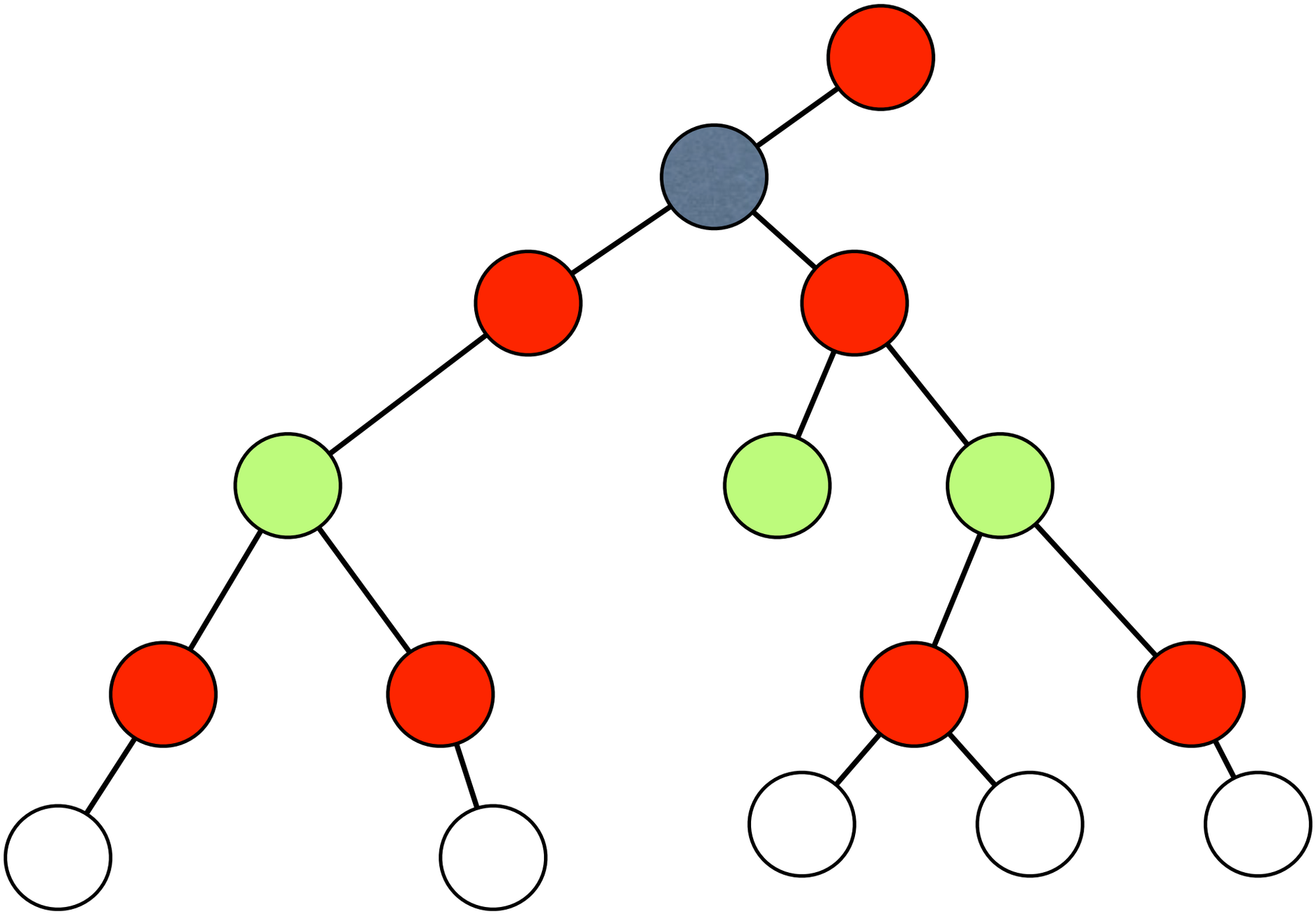

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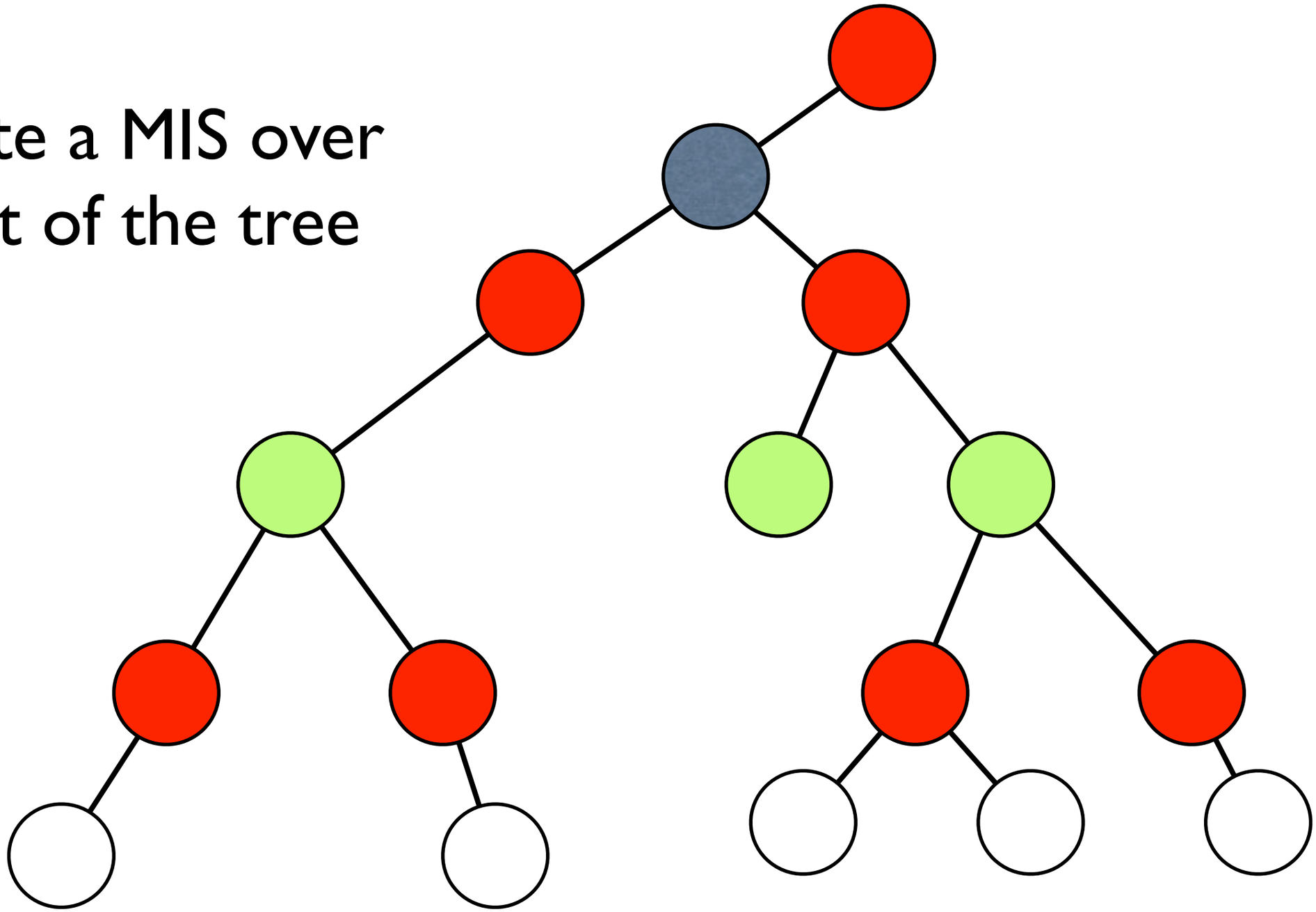
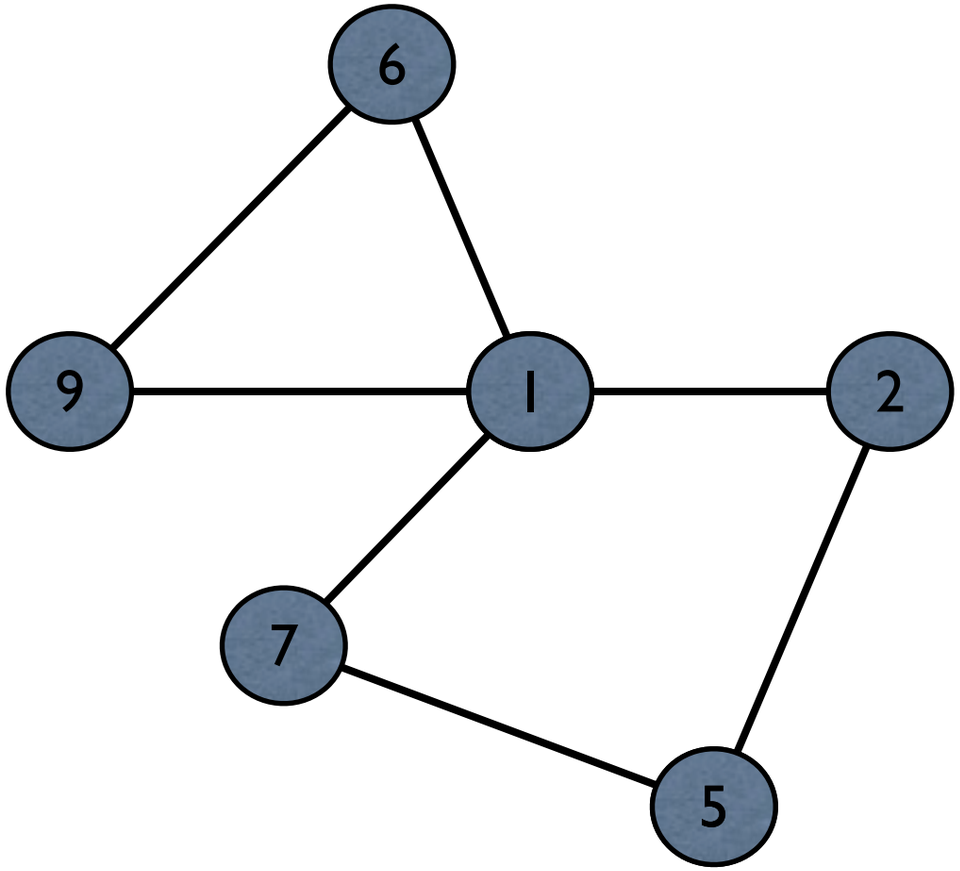


Connected DS



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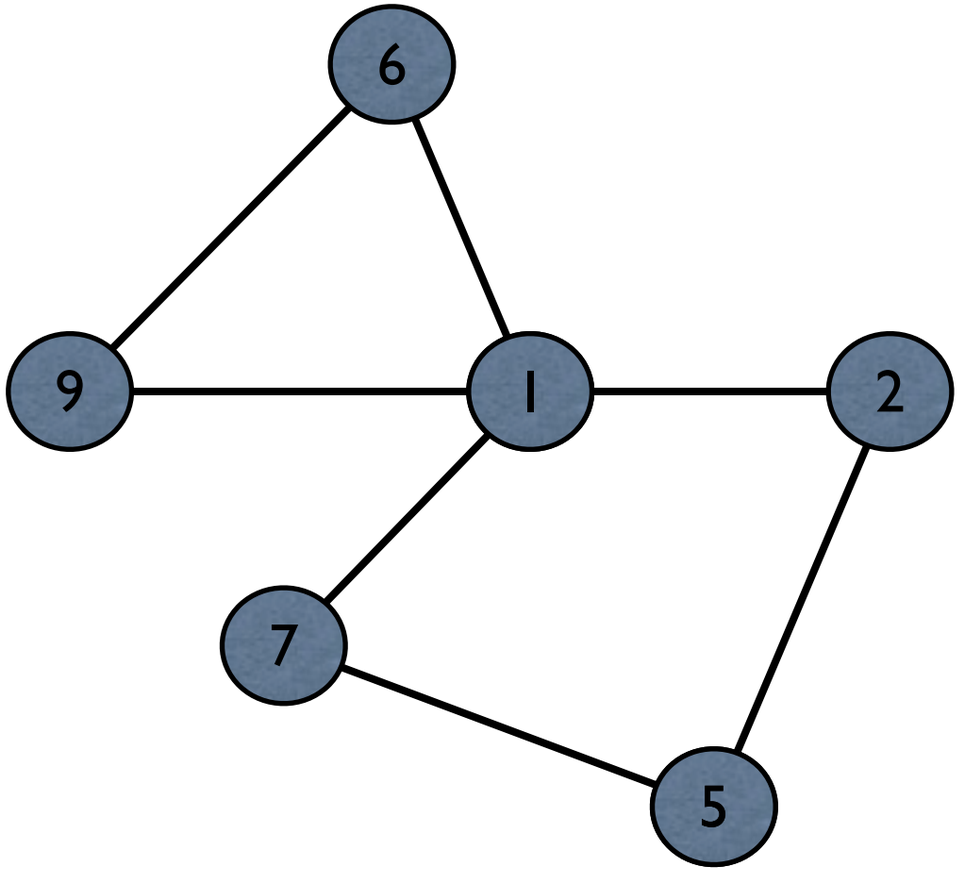

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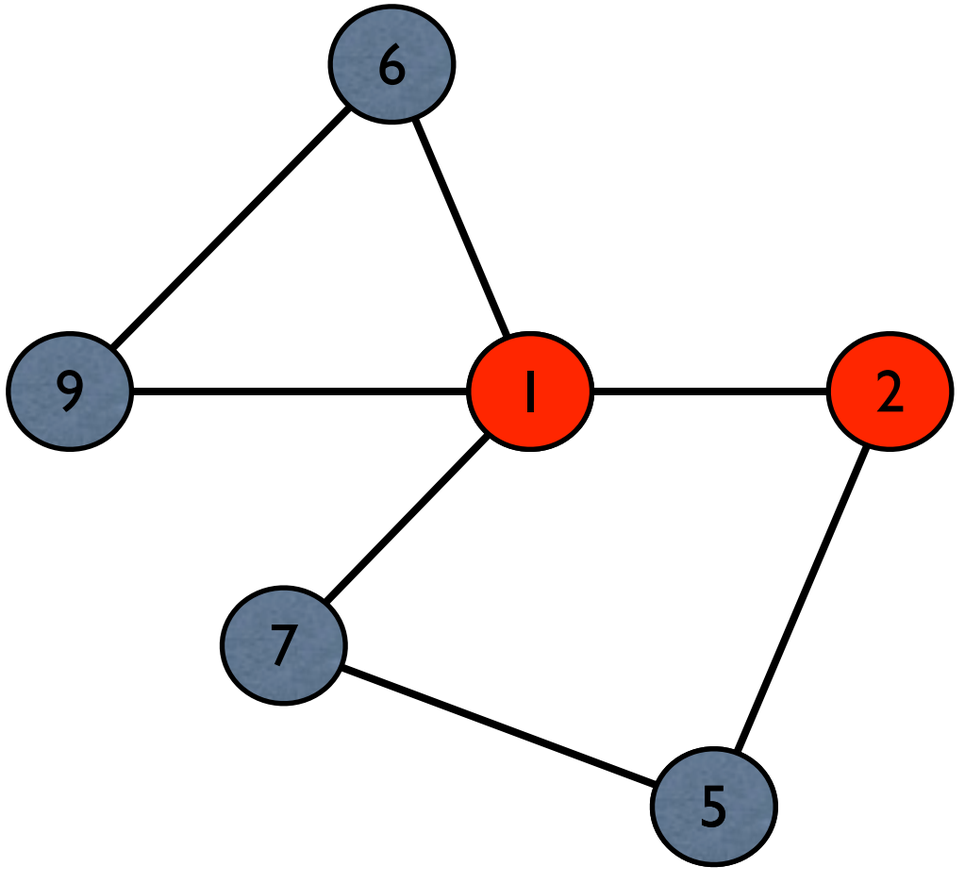
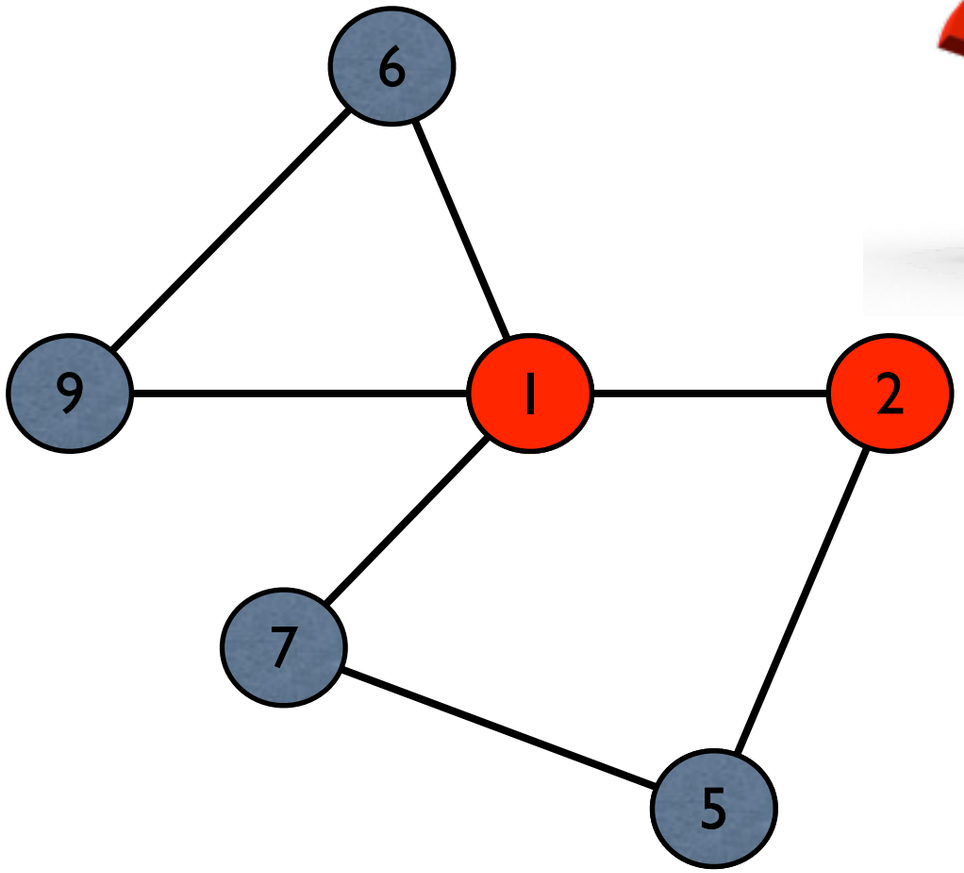
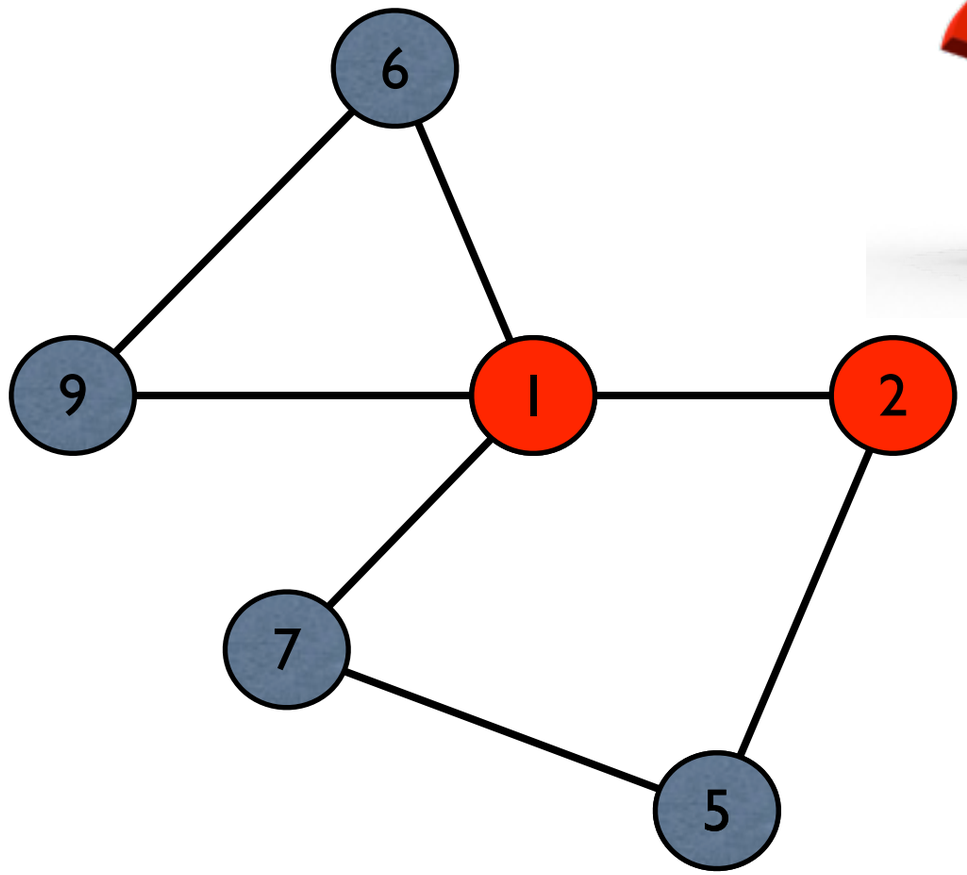


Connected DS



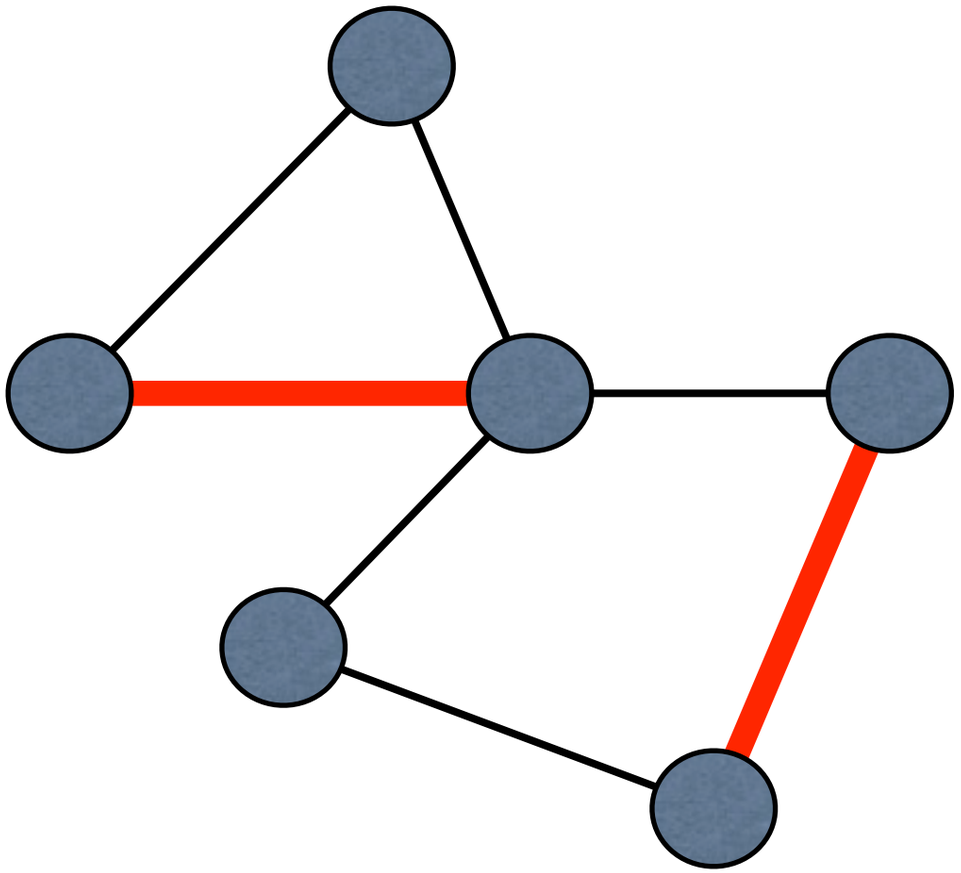
Connected DS



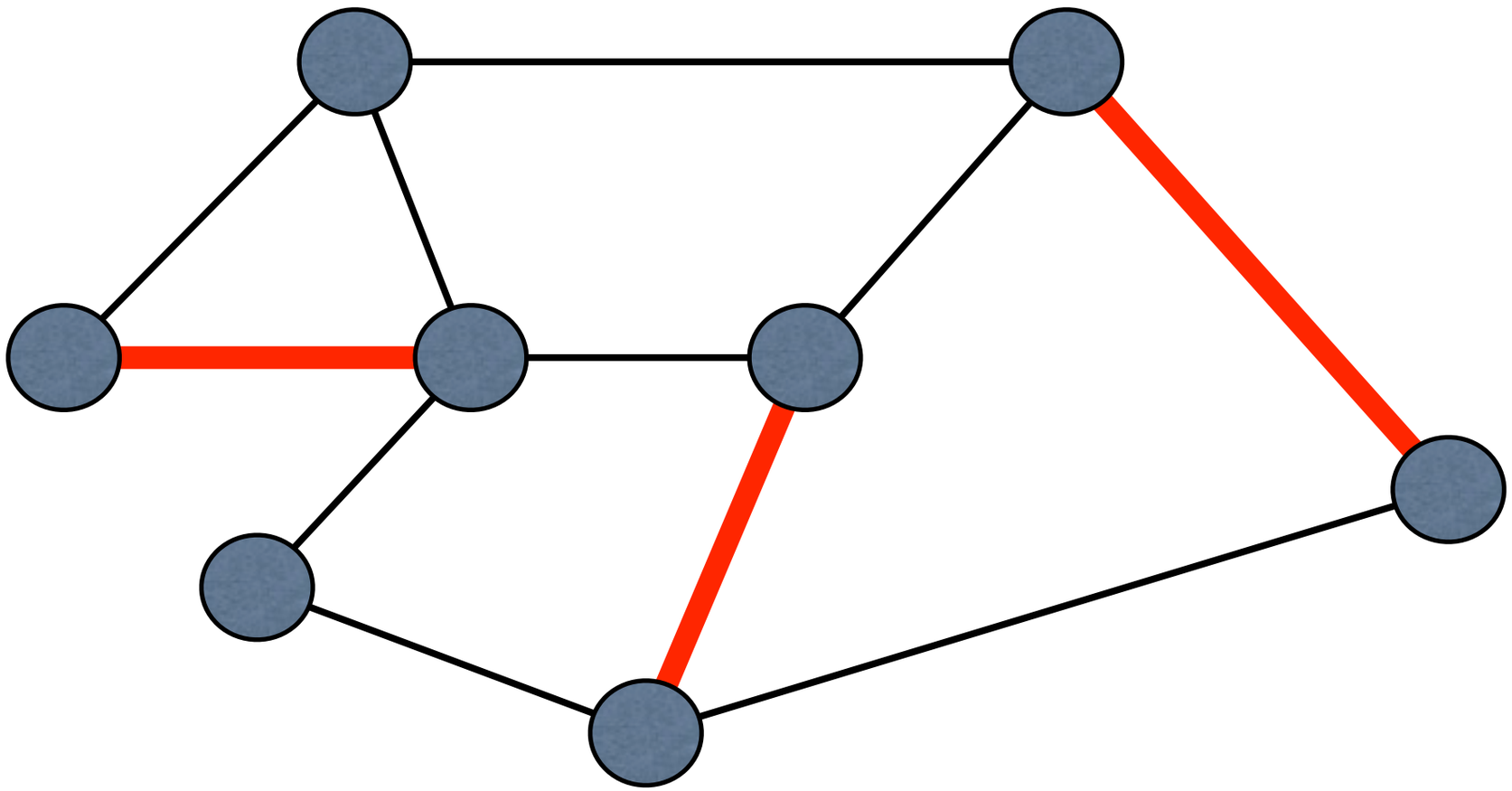
Matching

Matching

- Given a graph G=(V,E), a **matching** is a subset of edges, M, such that no two edges in M are adjacent in G.

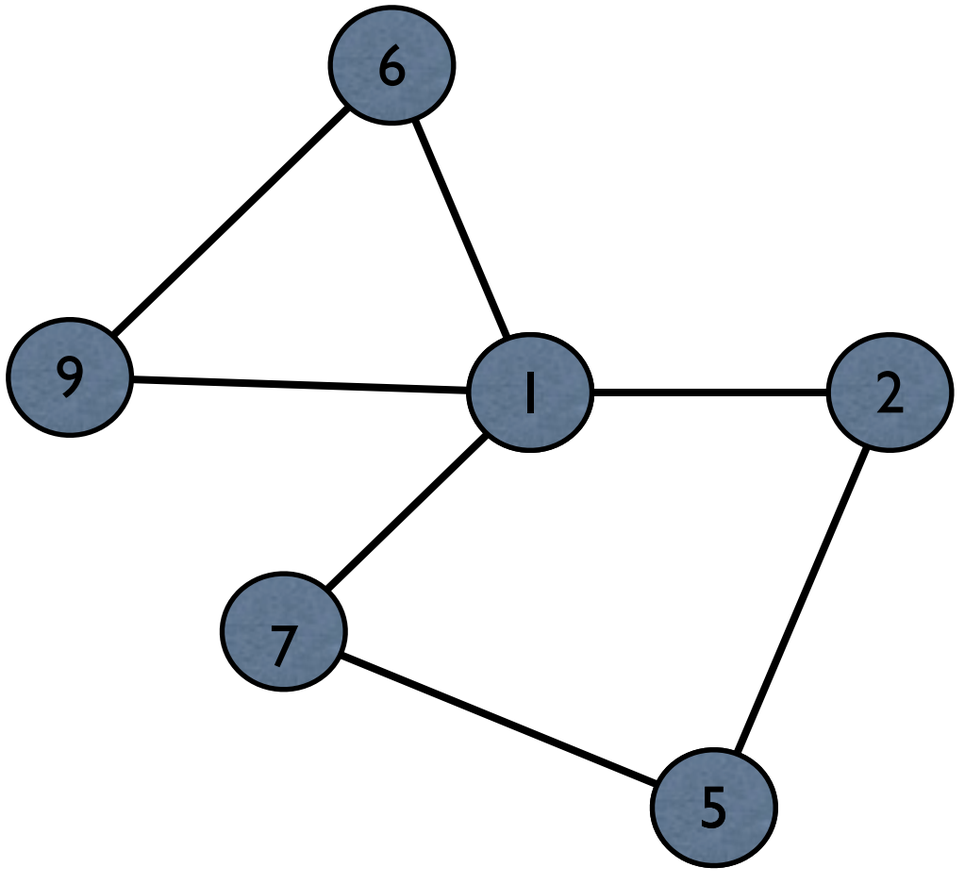


Maximal Matching



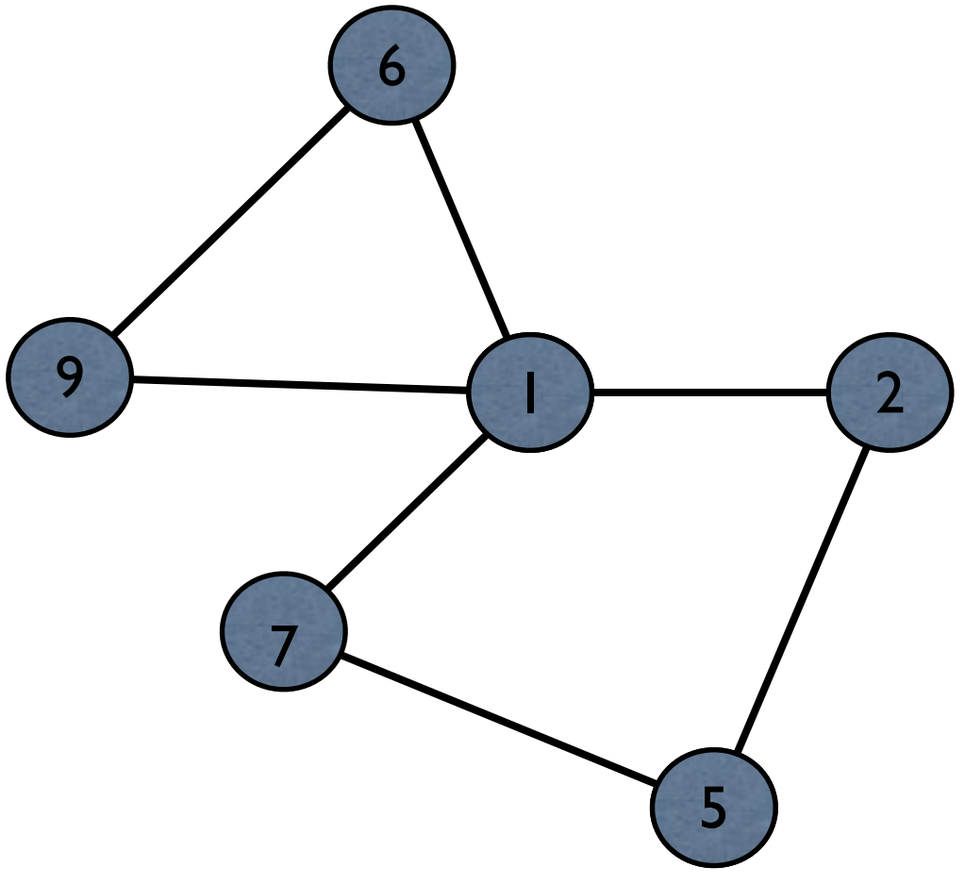
Reductions

From Coloring to MIS

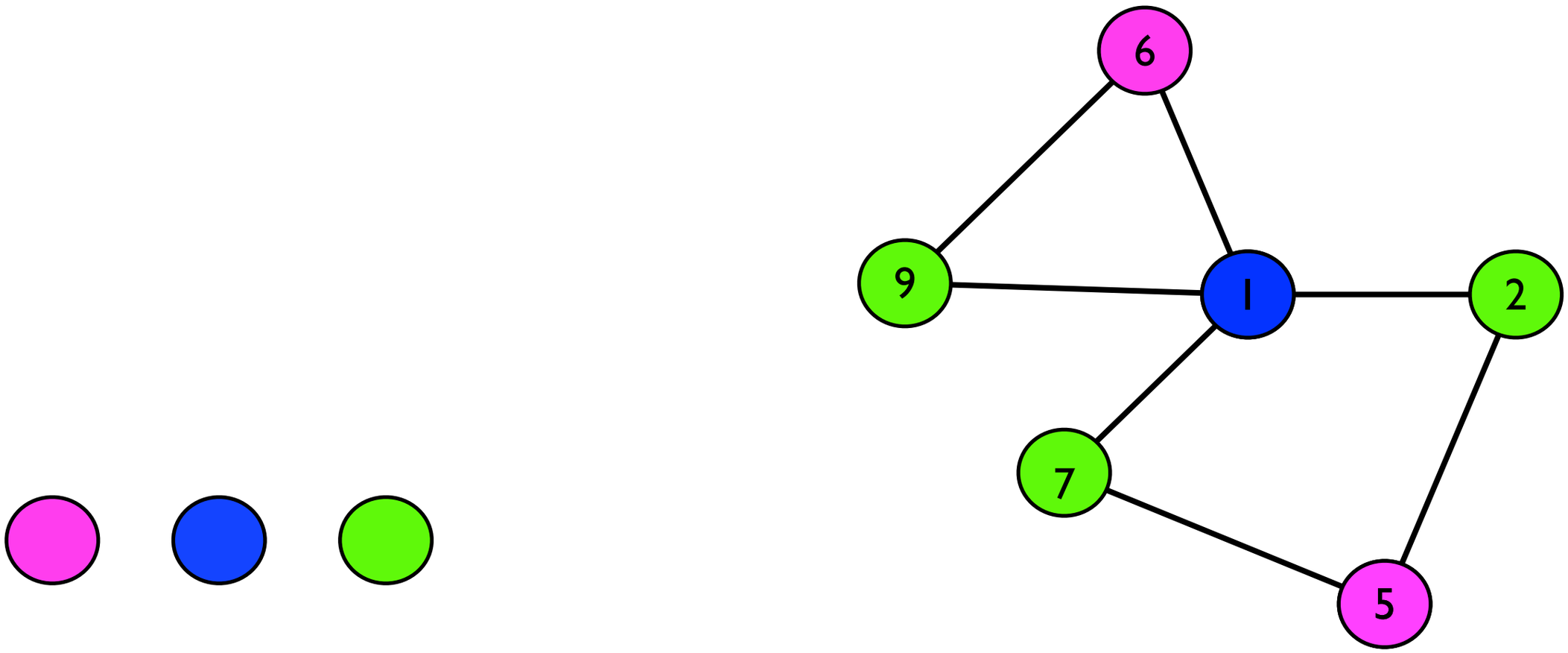


From Coloring to MIS

- Start with a valid coloring

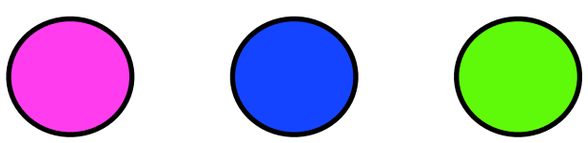


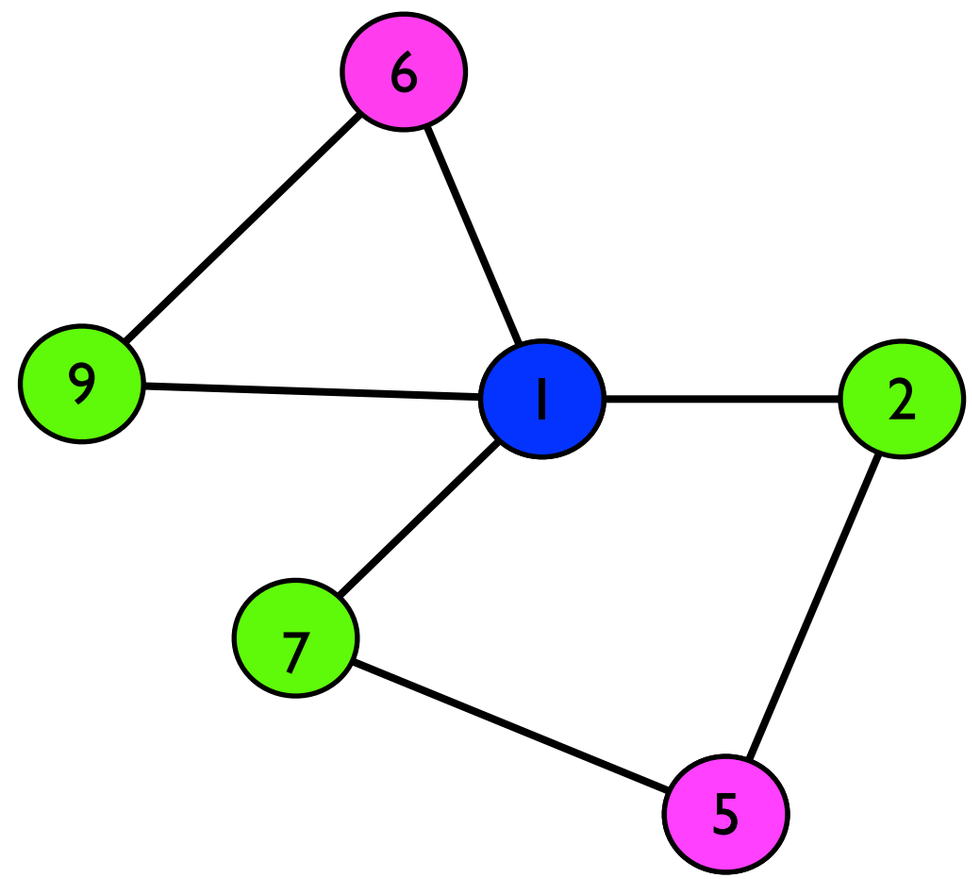
From Coloring to MIS



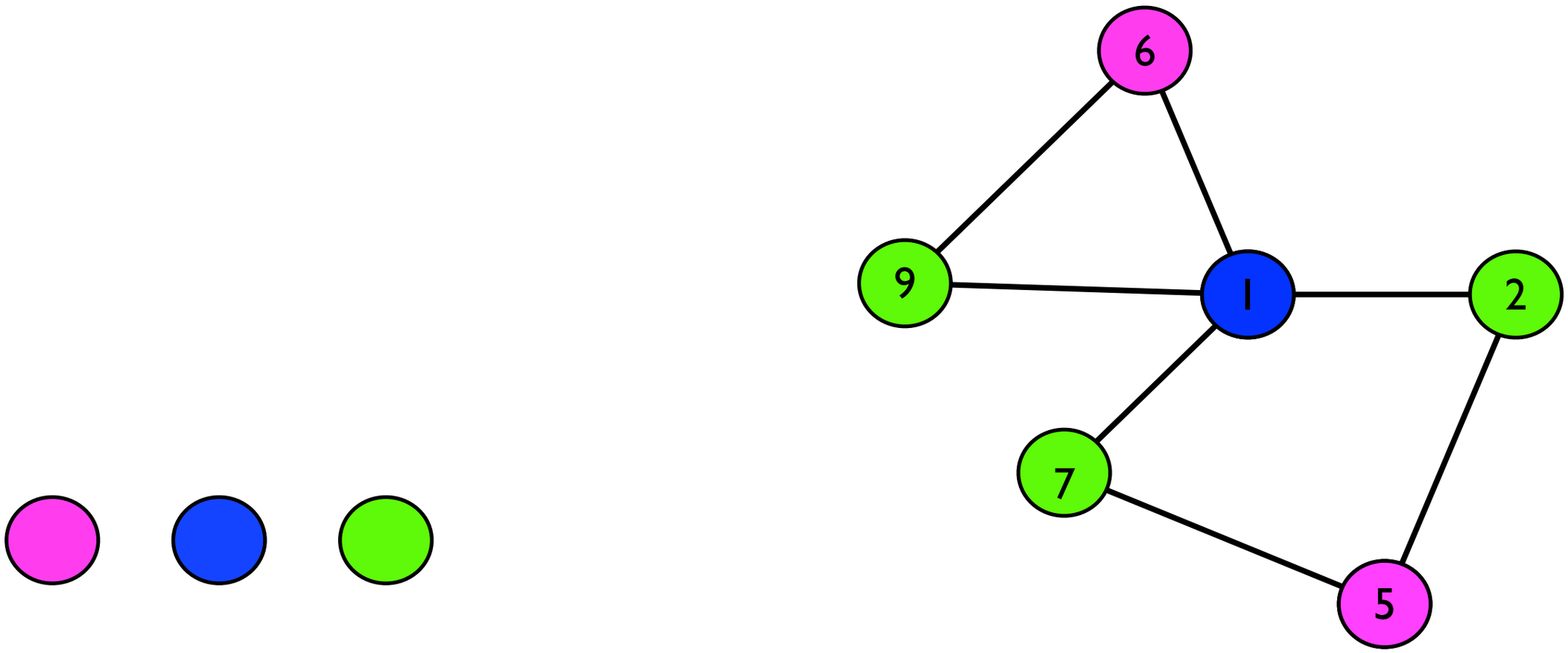
From Coloring to MIS

- Start with a valid coloring
- Assume a total order on colors



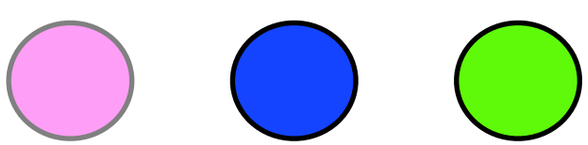


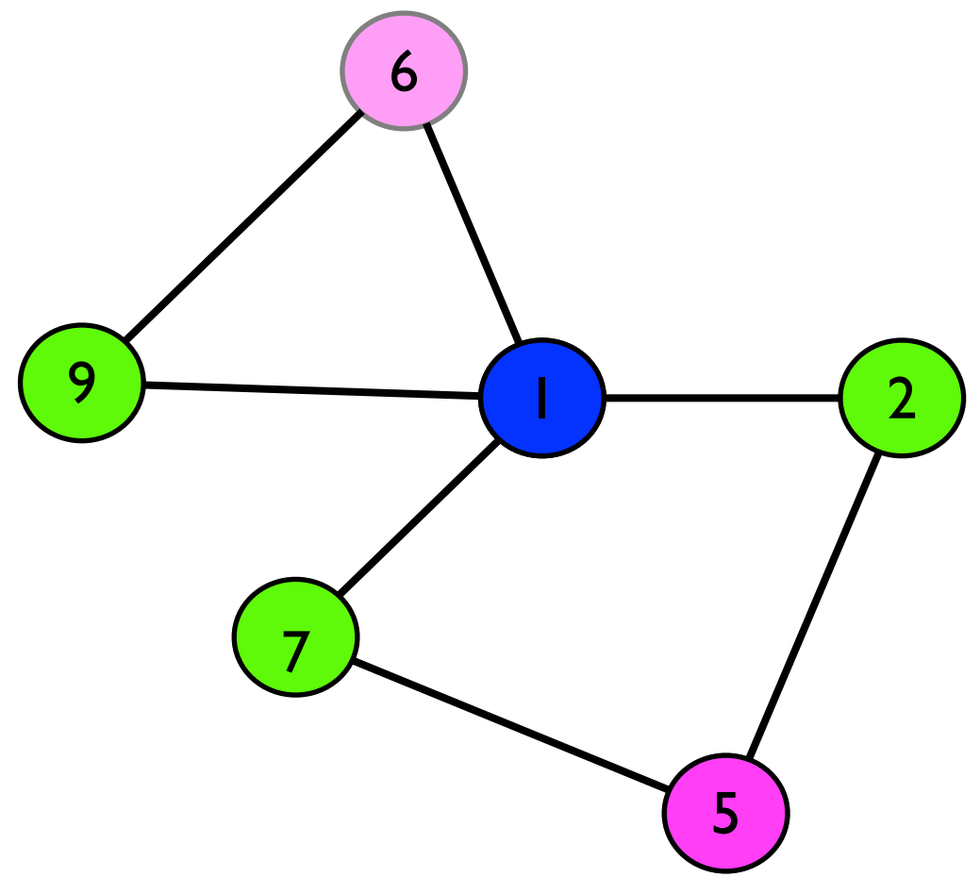
From Coloring to MIS



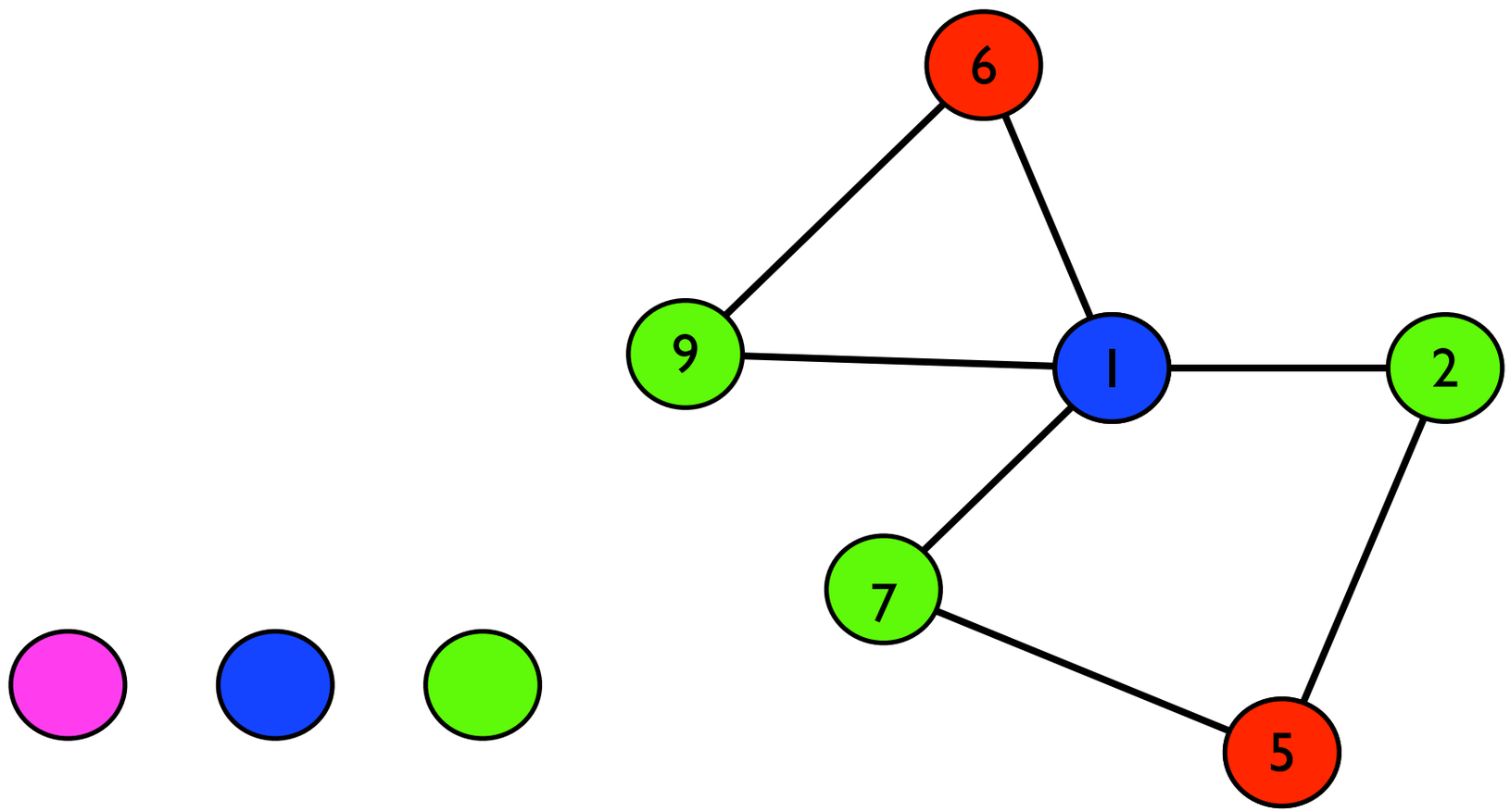
From Coloring to MIS

- Choose in MIS all nodes of first color



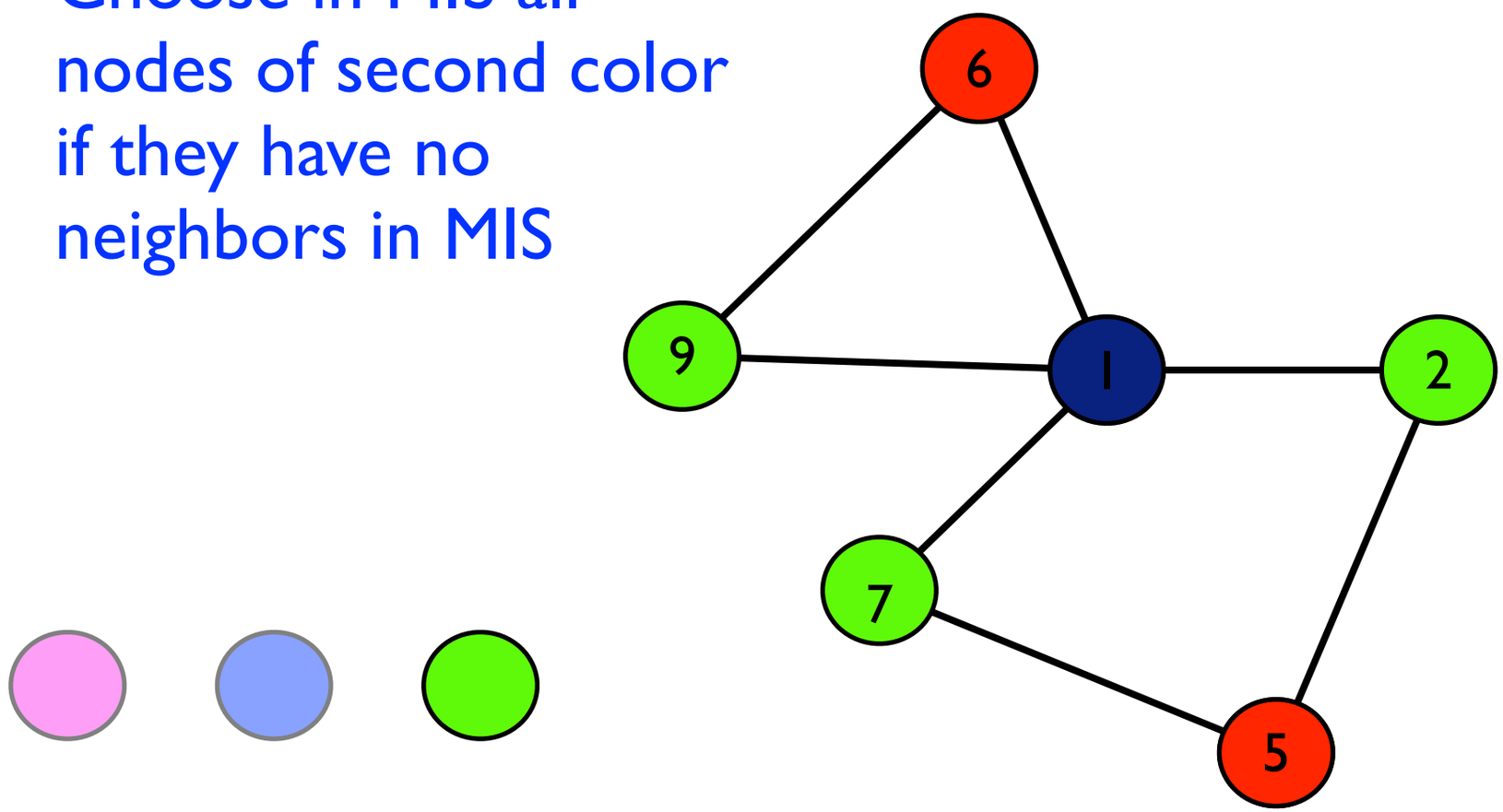


From Coloring to MIS

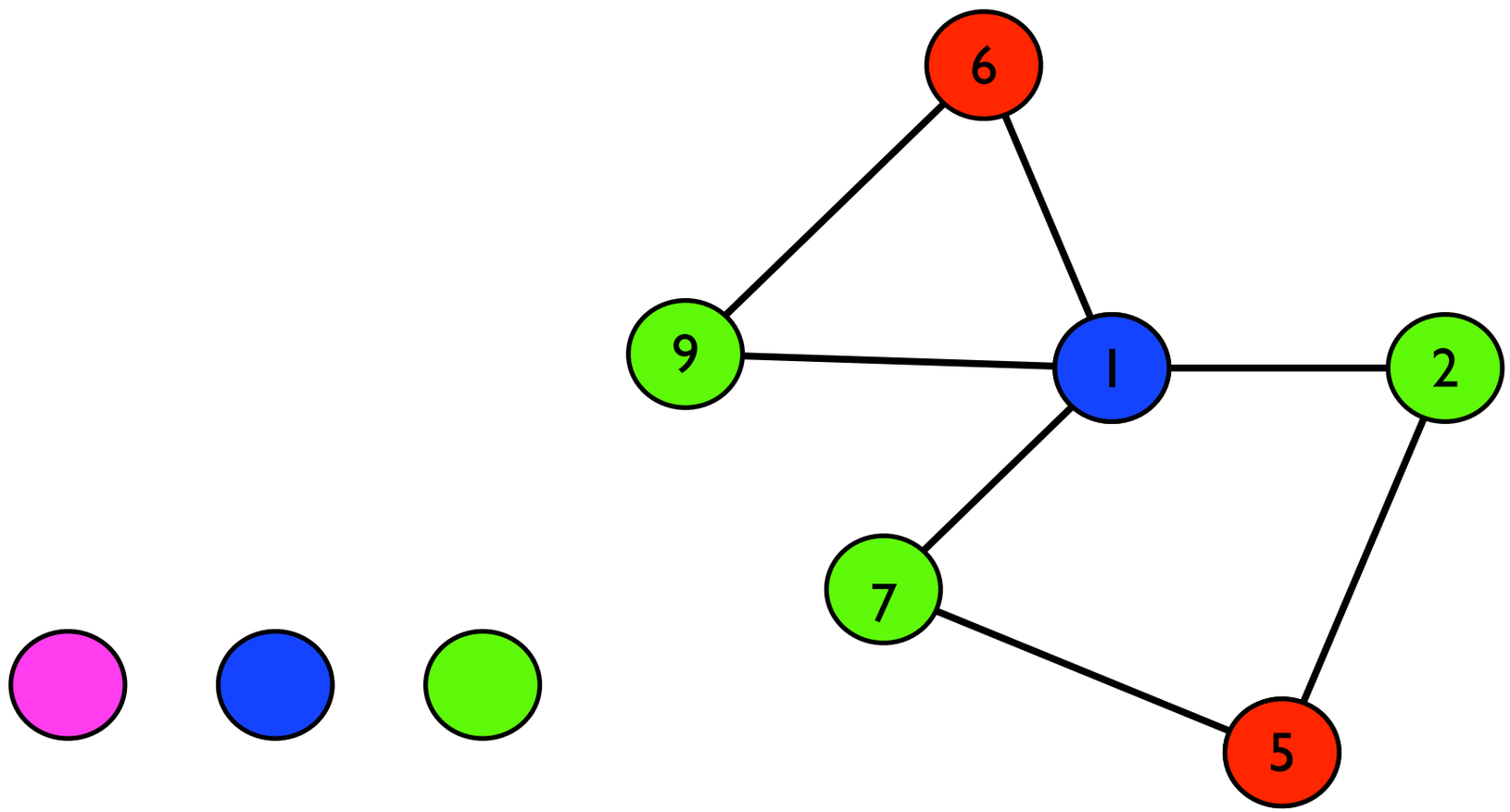


From Coloring to MIS

- Choose in MIS all nodes of second color if they have no neighbors in MIS

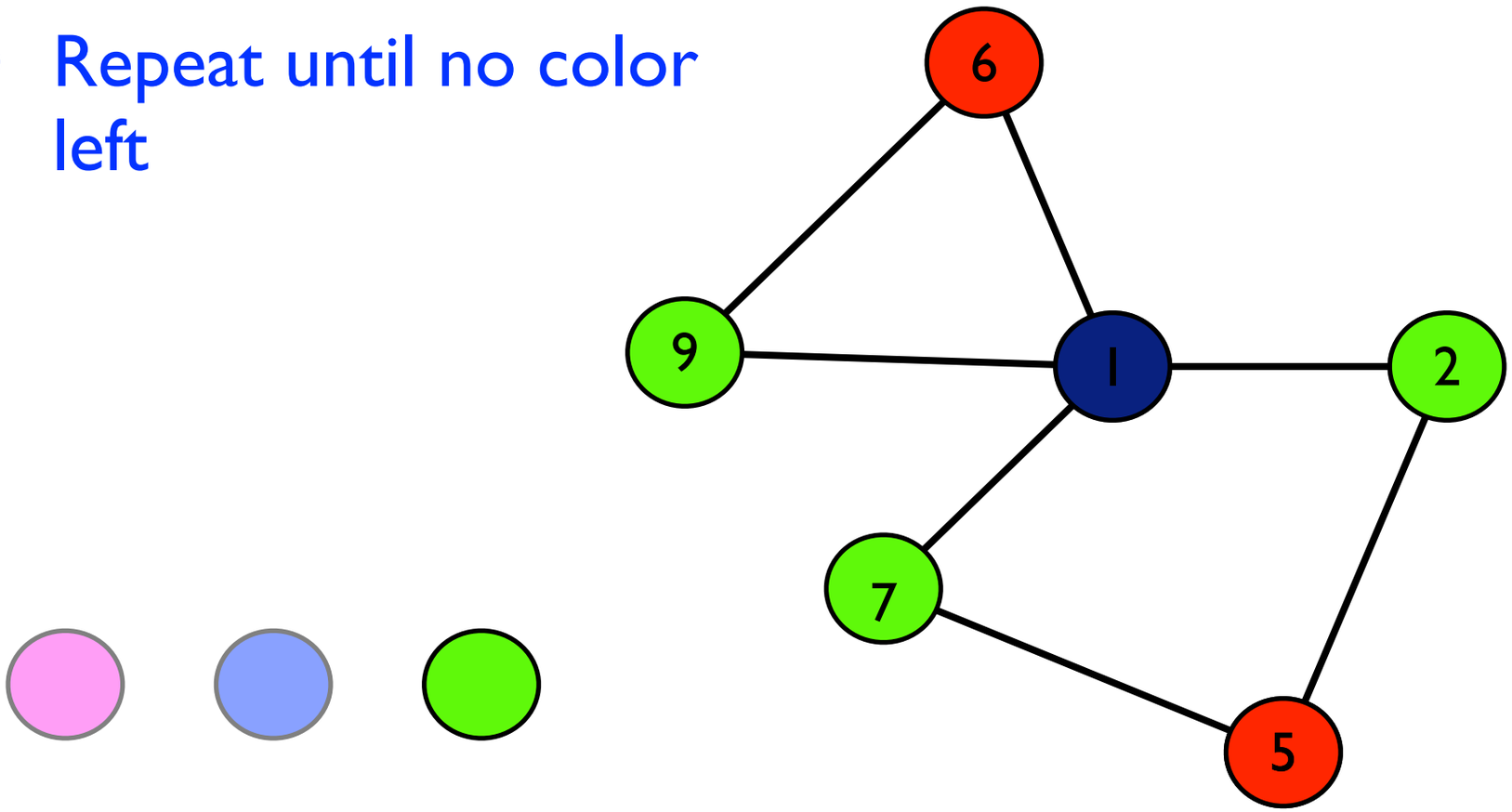


From Coloring to MIS



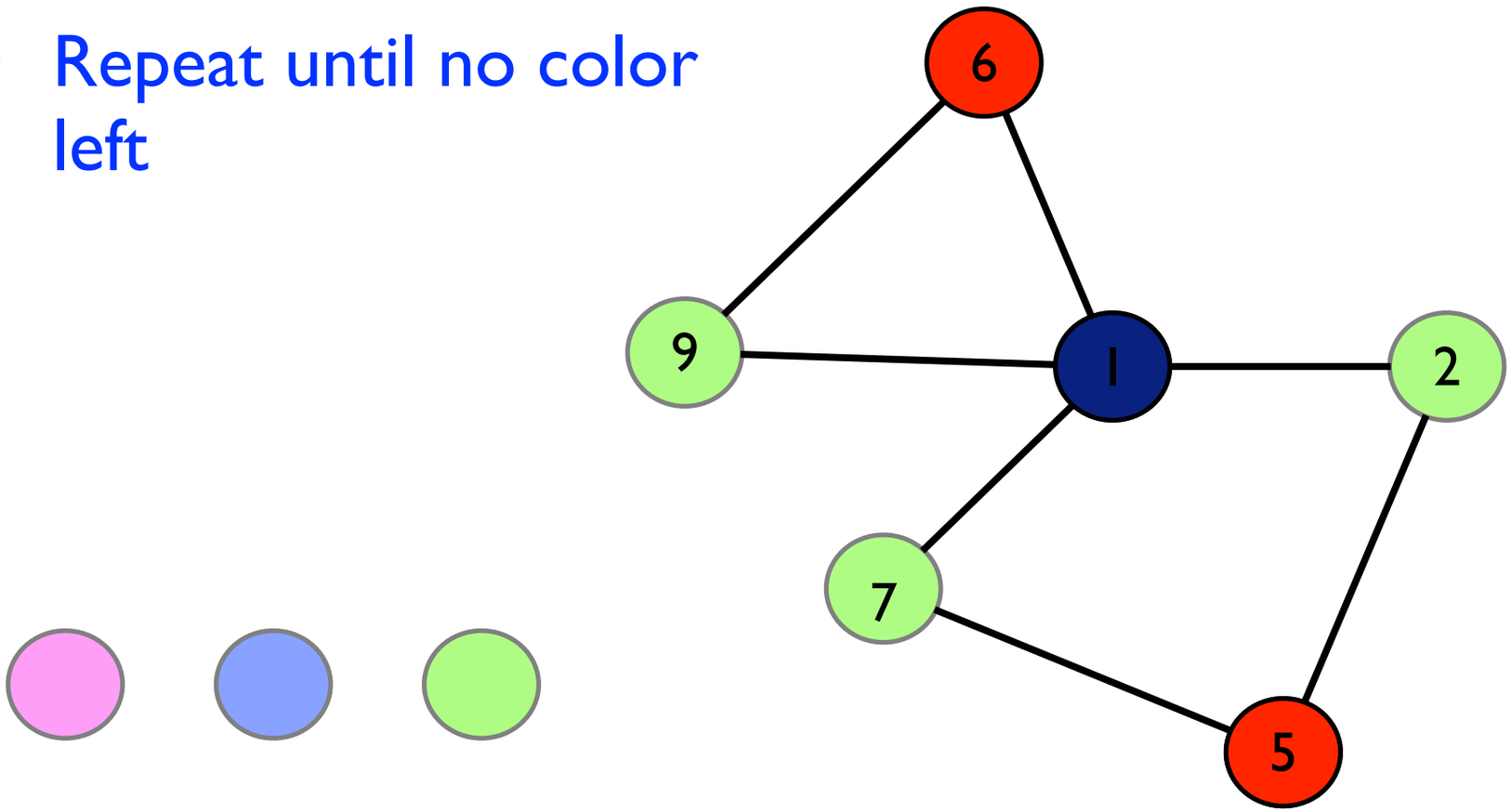
From Coloring to MIS

- Repeat until no color left

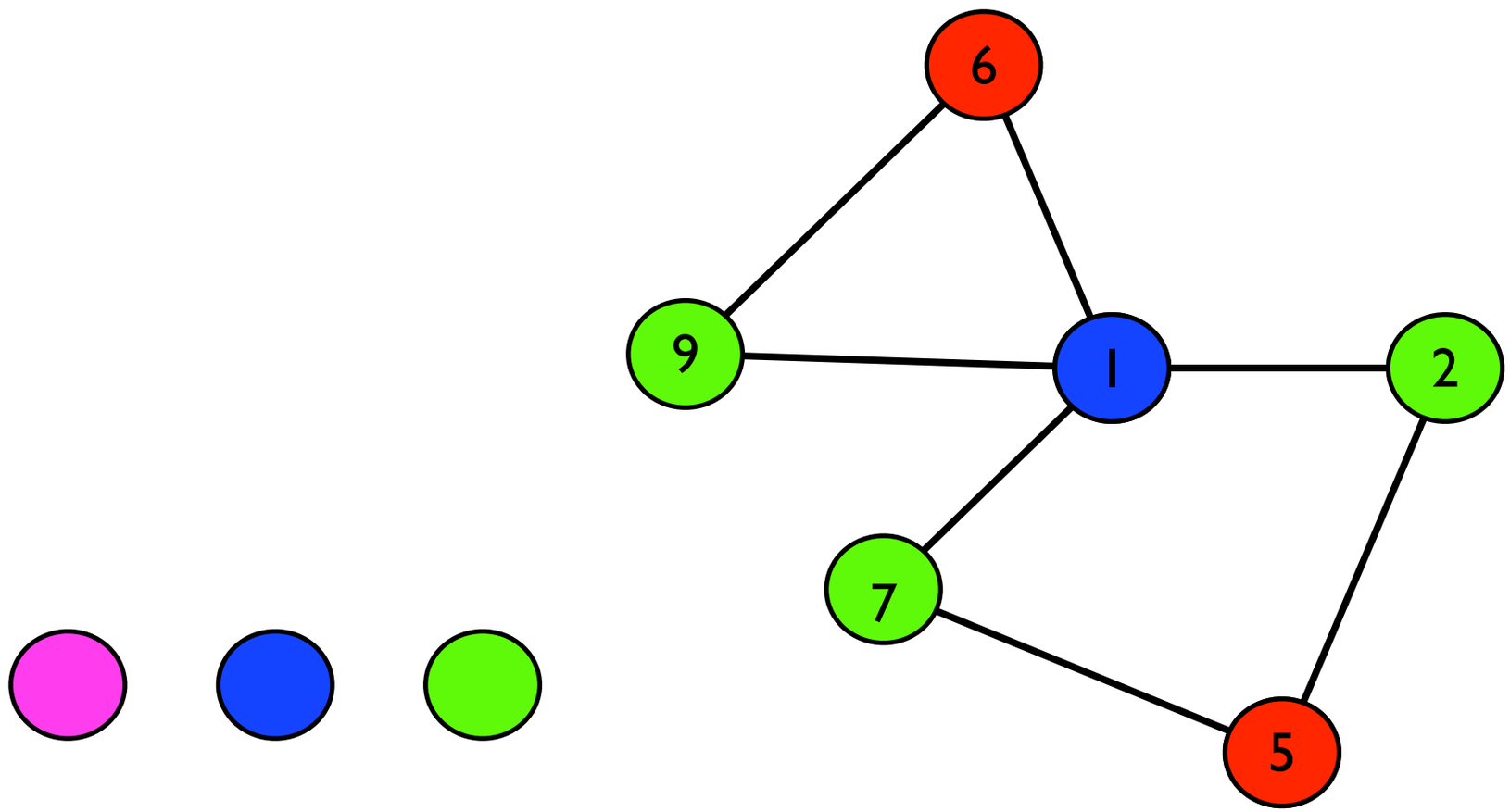


From Coloring to MIS

- Repeat until no color left

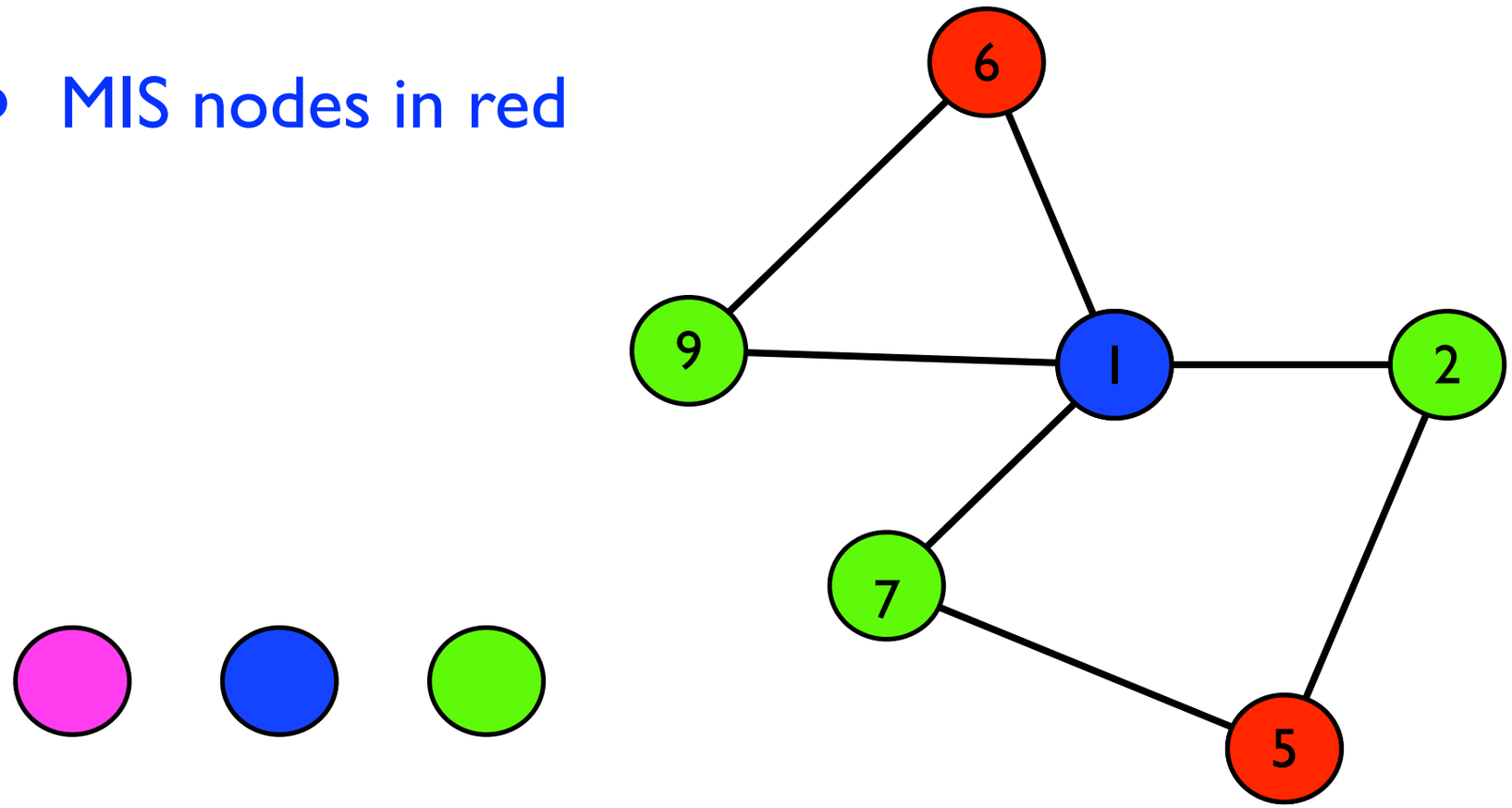


From Coloring to MIS



From Coloring to MIS

- MIS nodes in red



From Coloring to MIS

Theorem

Given a graph G and a coloring of G with f(G) colors in time T(G) it is possible to construct a MIS for G in time T(G)+f(G).

From Coloring to MIS

Corollary

There exists a deterministic MIS algorithms for trees and bounded degree graphs with time complexity $O(\log^*(n))$.

From Coloring to DS

Corollary

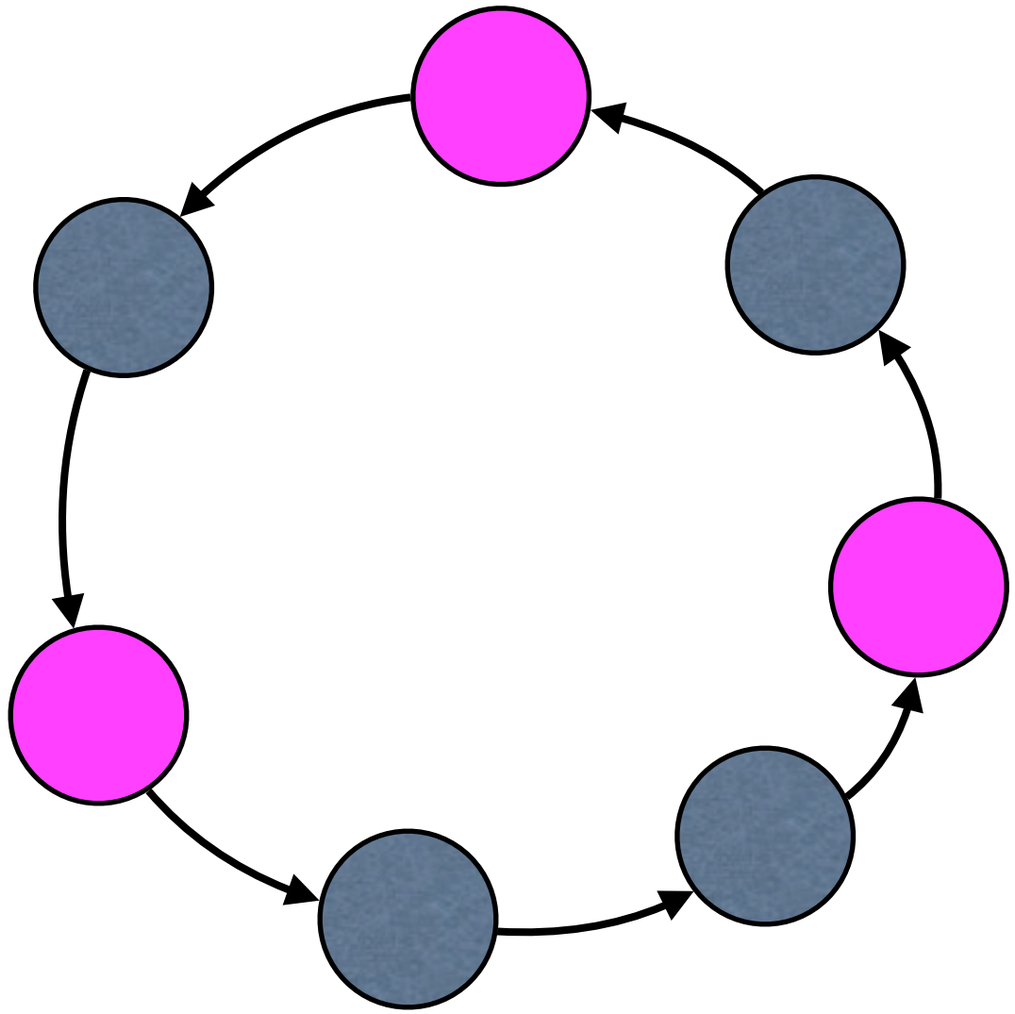
There exists a deterministic DS algorithms for trees with time complexity $O(\log^*(n))$.

From MIS to Coloring

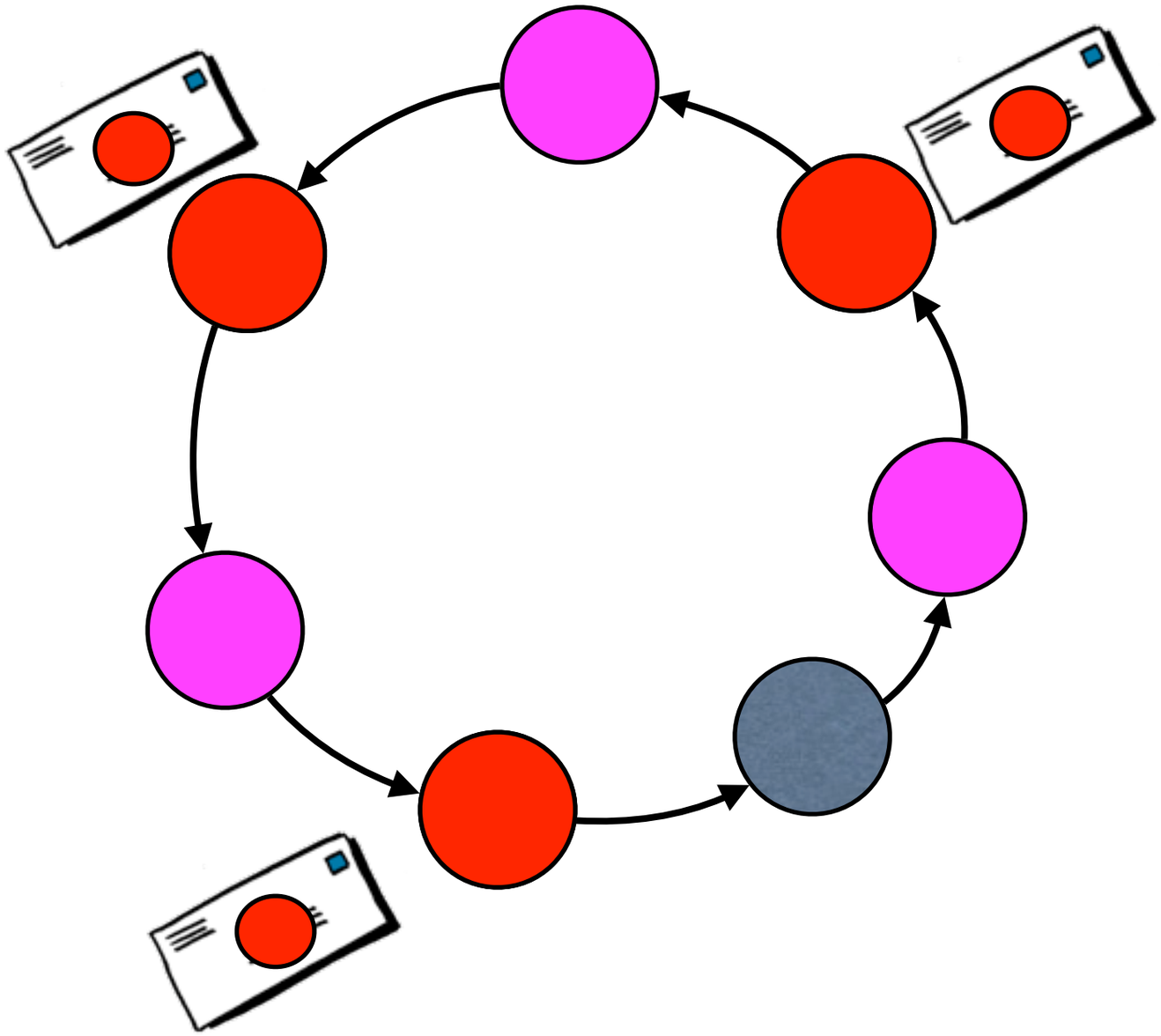
Theorem

Given a MIS on an oriented ring it is possible to 3-color the ring in one round.

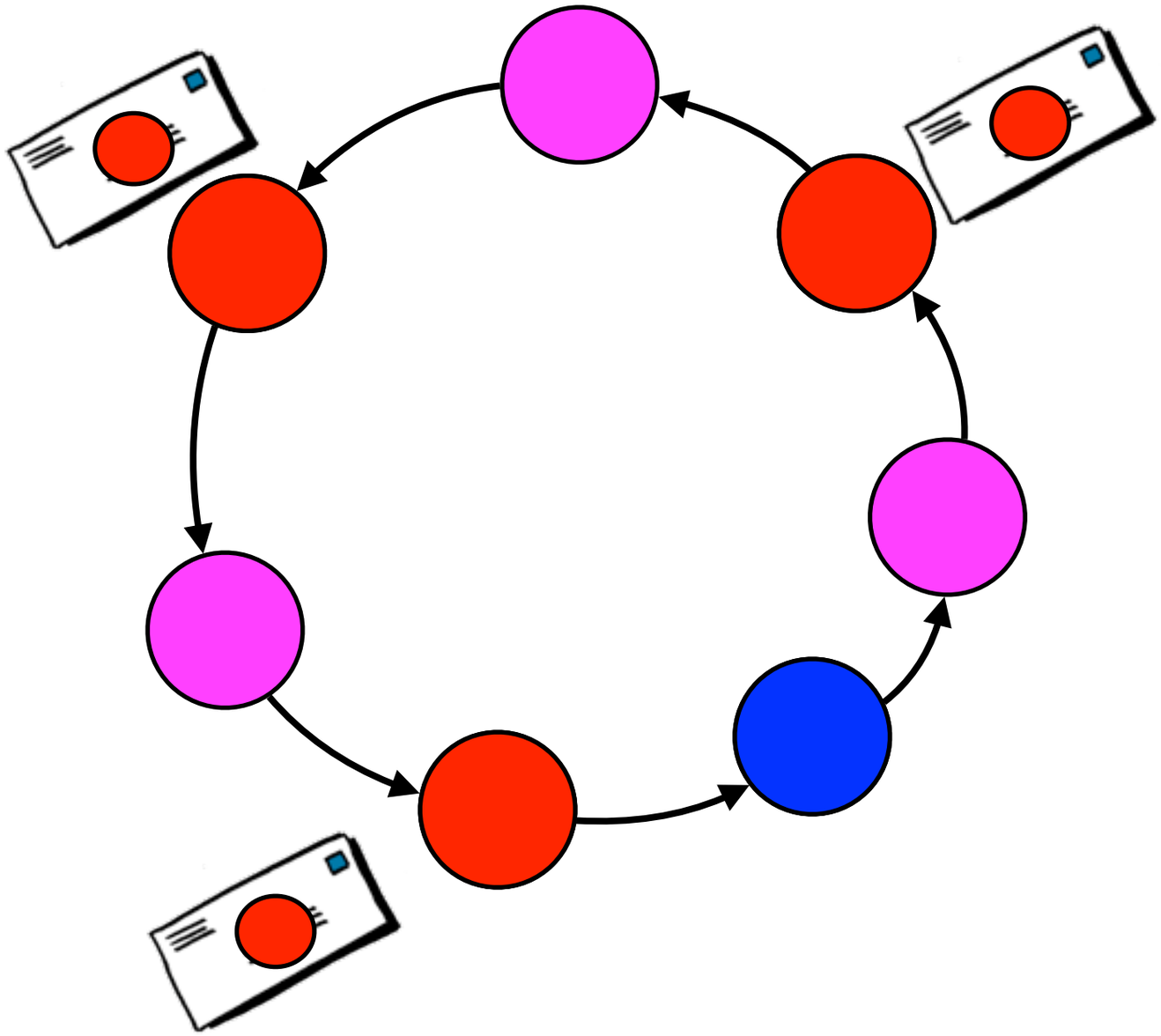
From MIS to Coloring



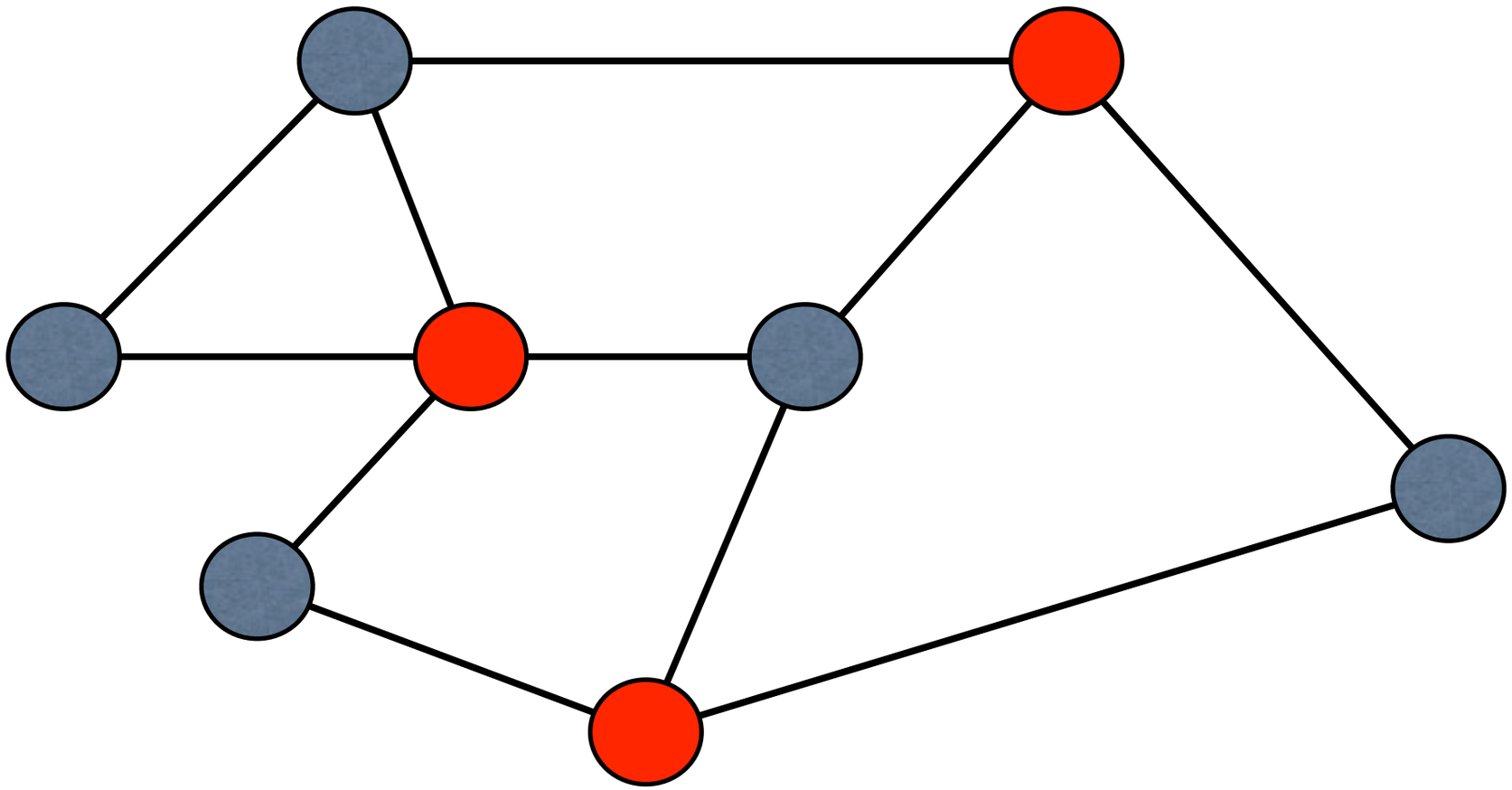
From MIS to Coloring



From MIS to Coloring



From MIS to Maximal Matching



From MIS to Maximal Matching