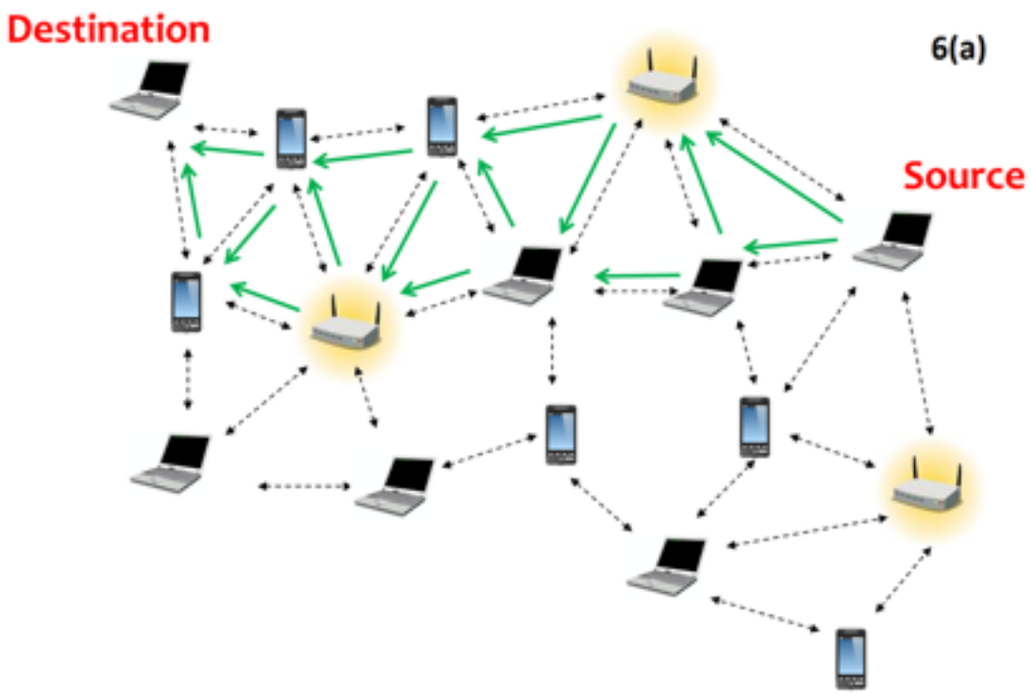
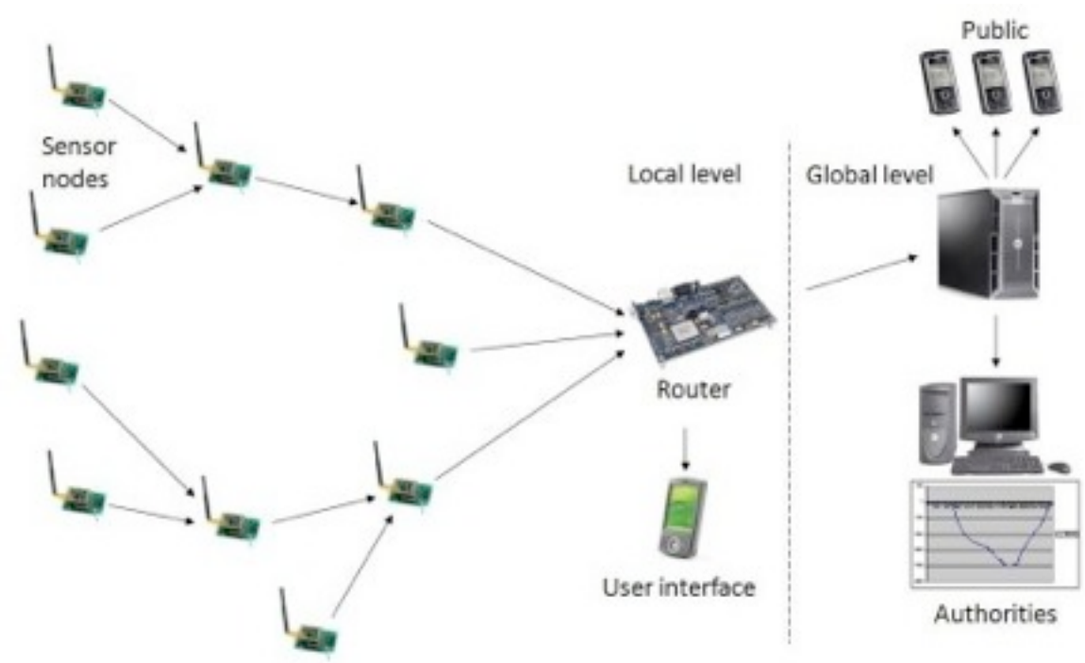
Trees & Routing (1)
BFS

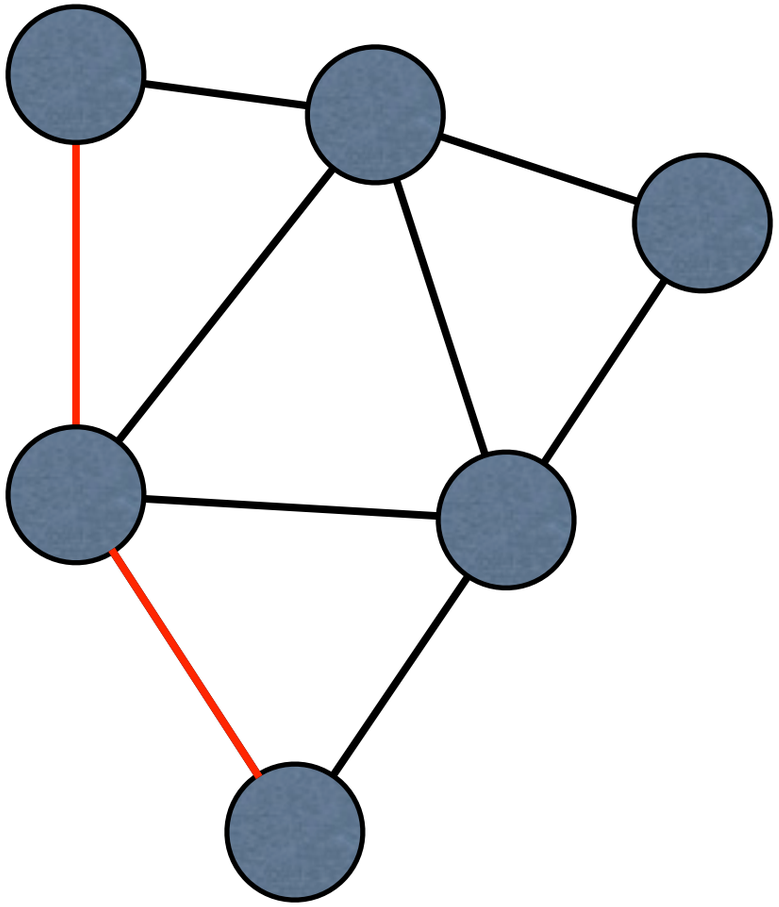
Why (spanning) trees ?





Spanning Tree

- Given a graph G=(V,E), a **Spanning Tree** of G is an acyclic subgraph, spanning all the nodes in V.



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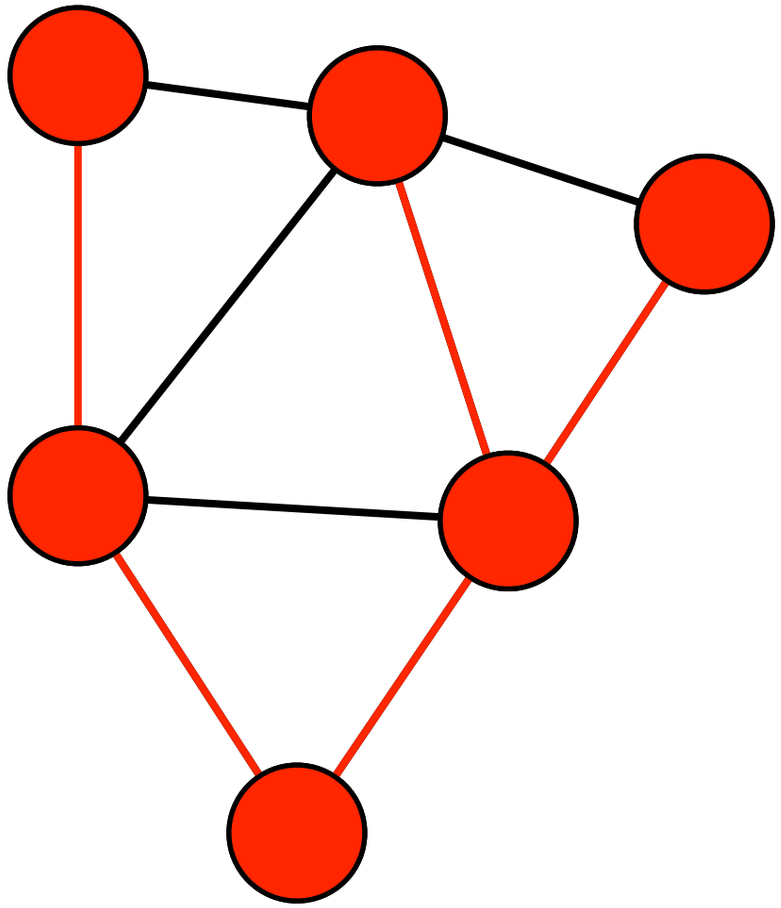
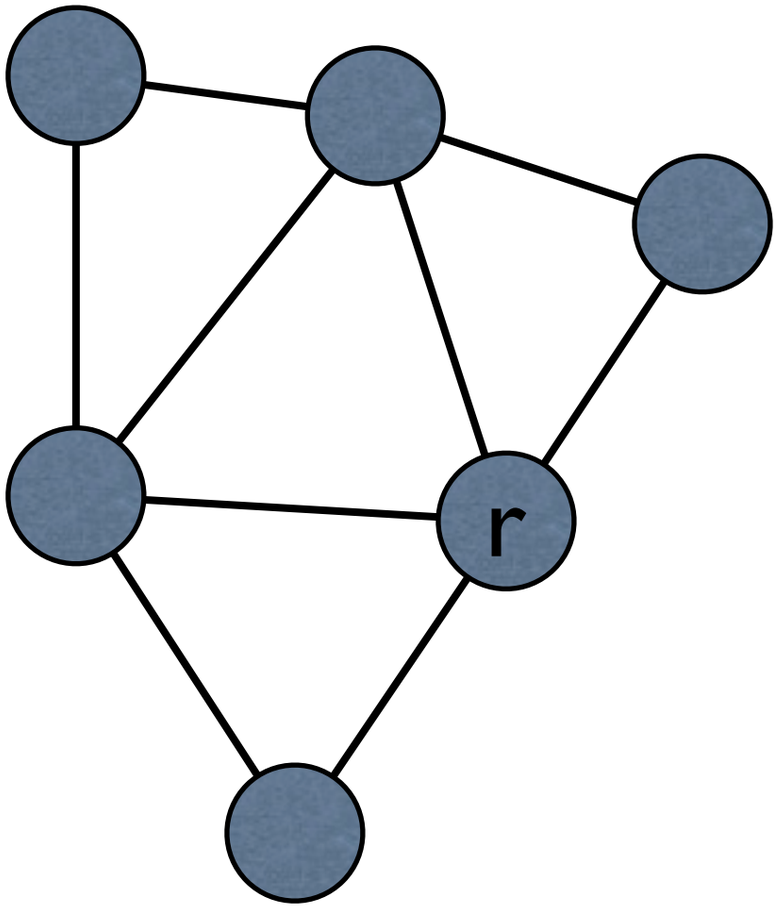


Breadth-first Spanning Tree (BFS)

- Given a graph G=(V,E) and a node r in V, a **BFS** tree of G rooted at r is a spanning tree of G such that for any node v the path leading from the root to v in the tree is of the minimum lenght possible.



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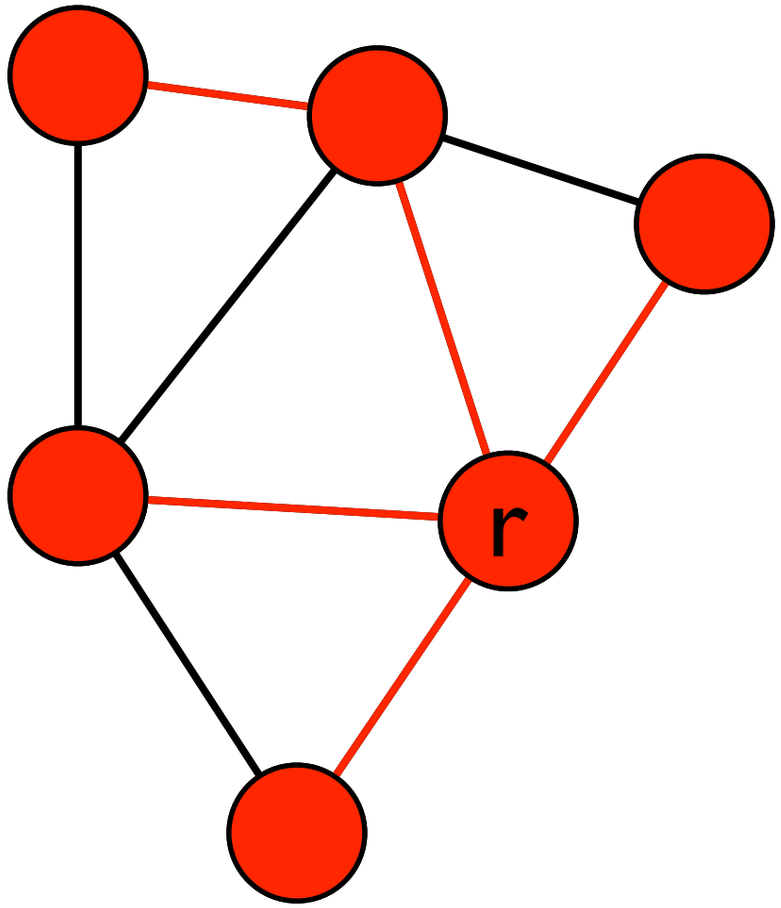
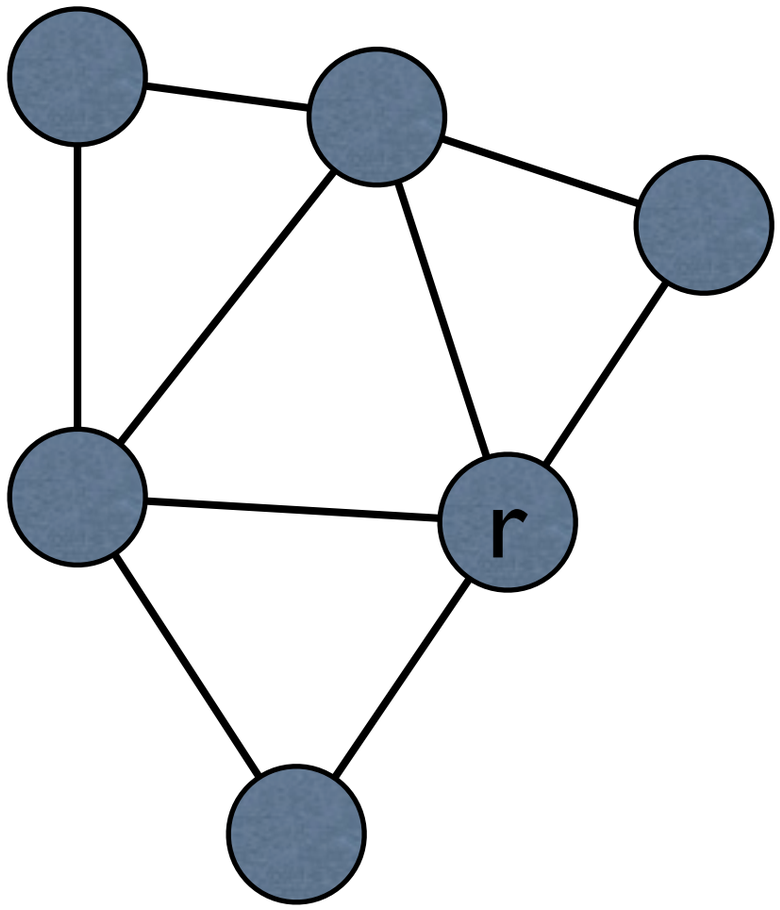


BFS Construction

BFS - strategies

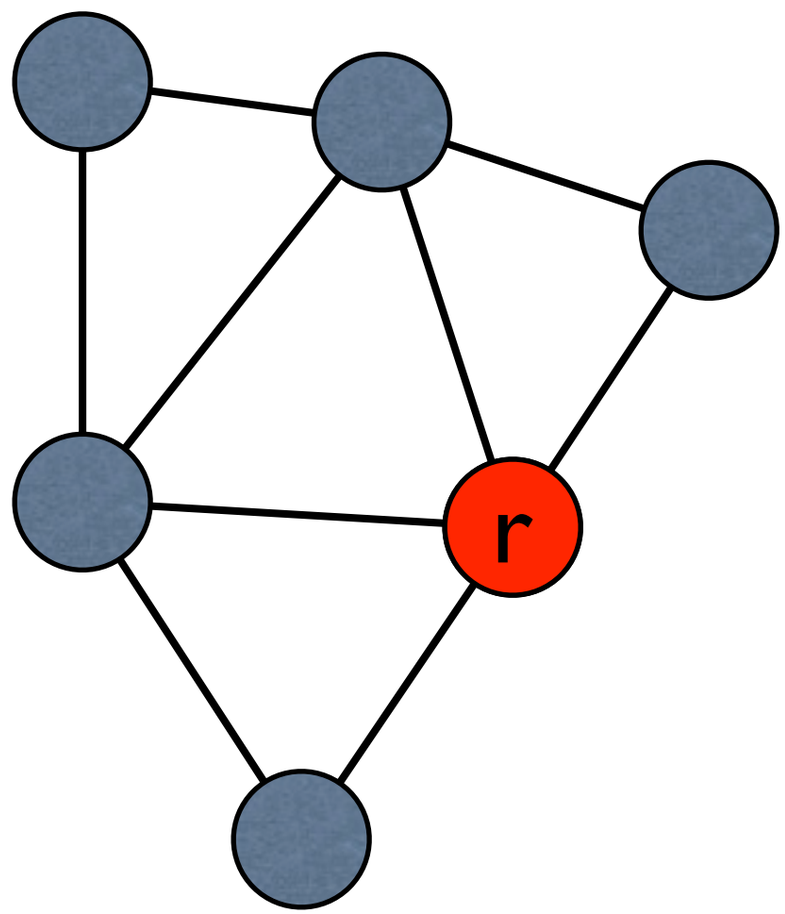
- «link state» (Dijkstra)
  - grow on the border
- «distance vector» (Bellman-Ford)
  - compute shortest paths

Dijkstra BFS

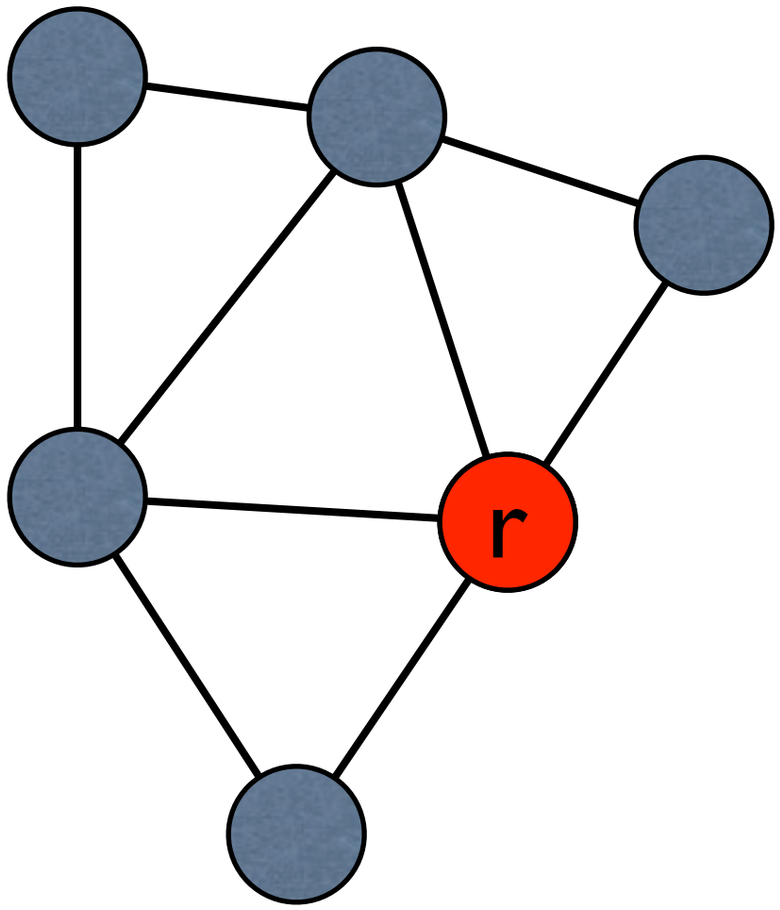


Dijkstra BFS

root, r, broadcasts in the curent tree, T, «pulse»

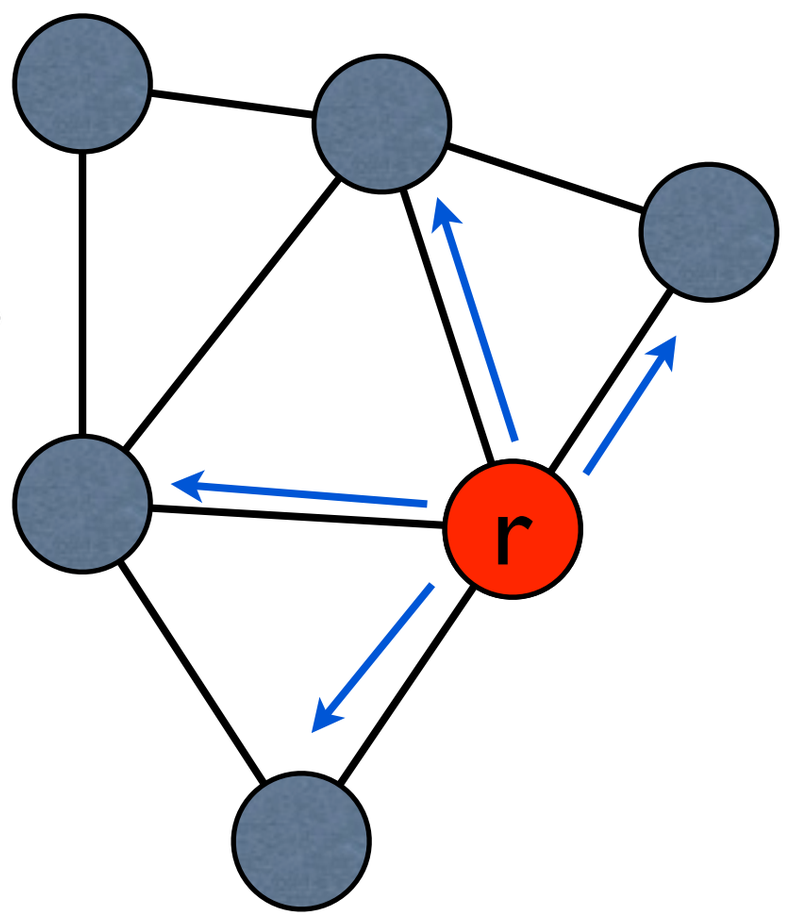


Dijkstra-like BFS



Dijkstra-like BFS

upon reception of «pulse», leafs of the curent tree send «layer» to neighbors (except to parents)



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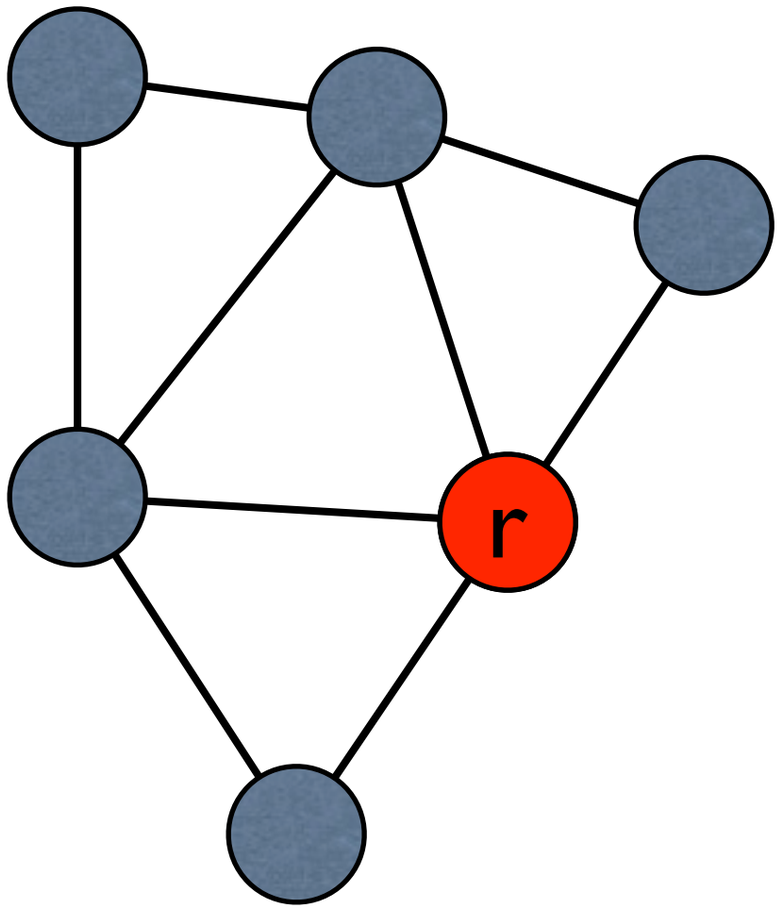
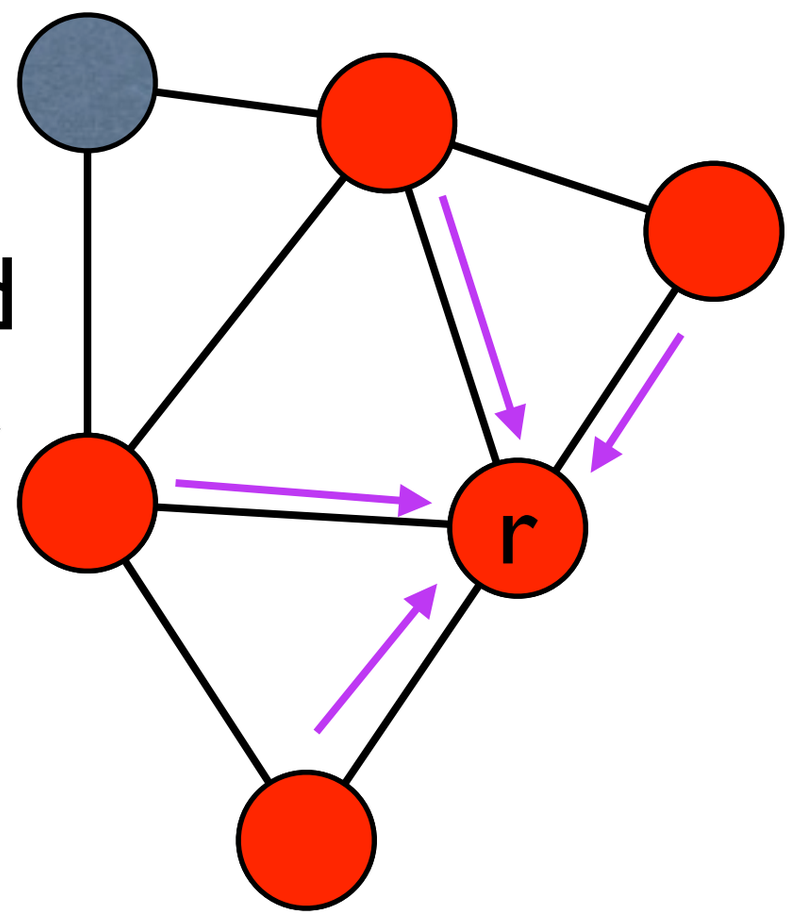


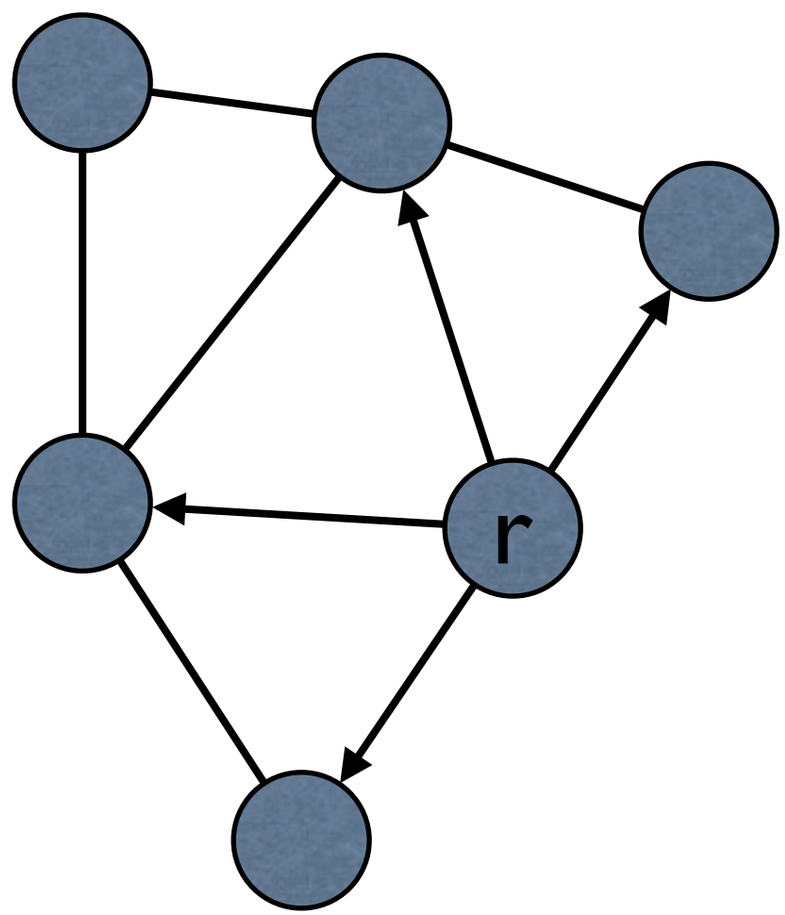
Dijkstra-like BFS

upon reception of the first «layer» a node choses the sender as parent and sends back an «ack»



Dijkstra-like BFS

upon reception of an «ack» a node register the sender as its child



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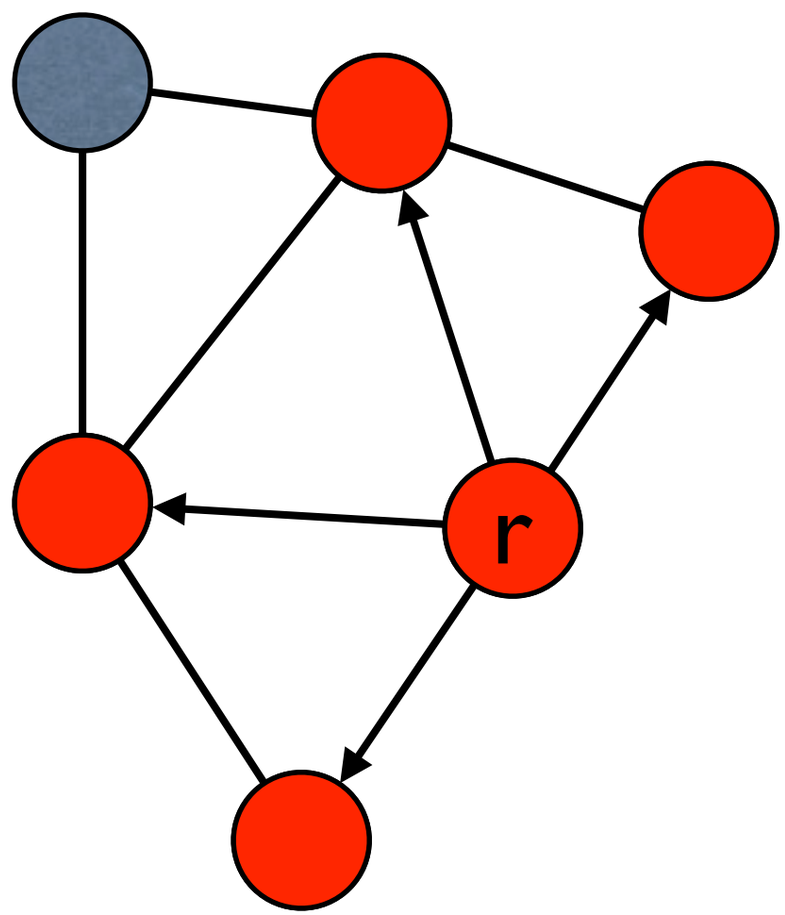

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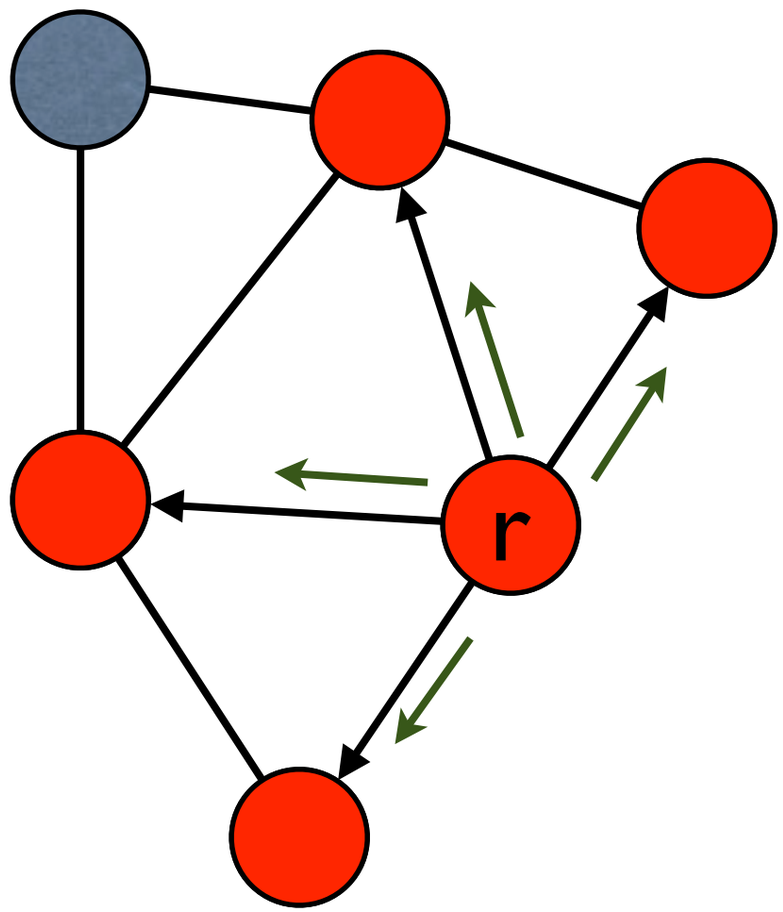

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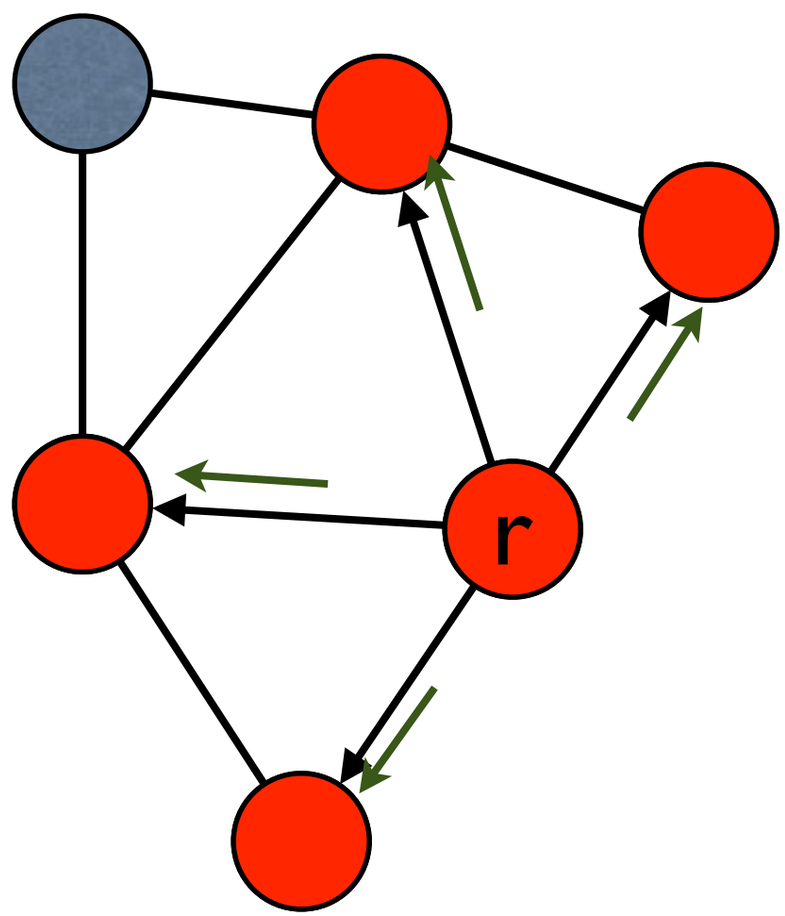

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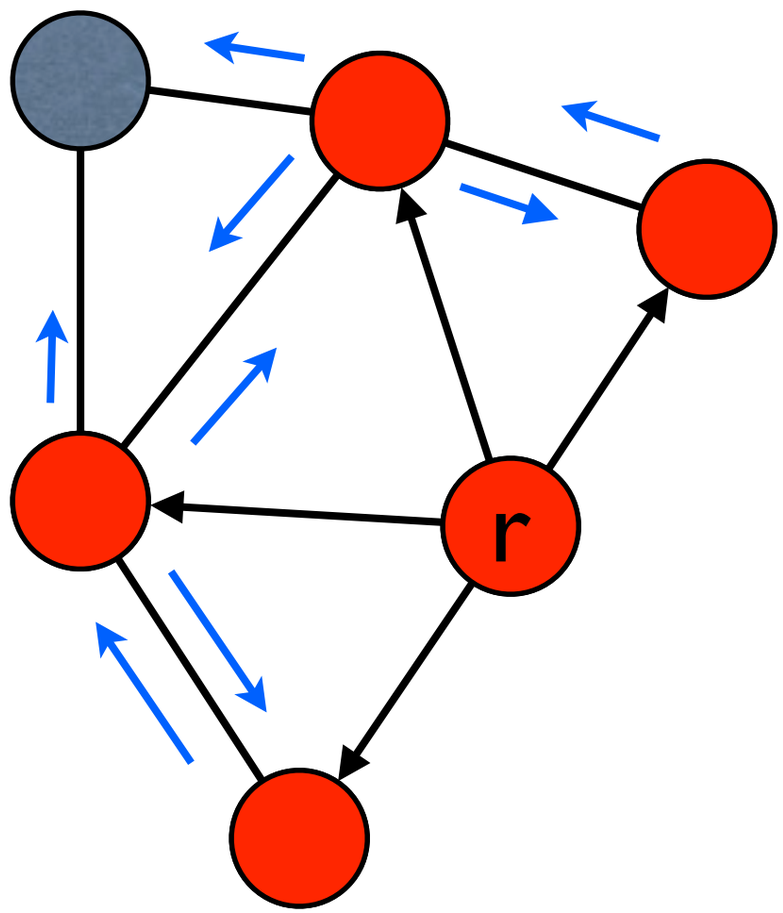

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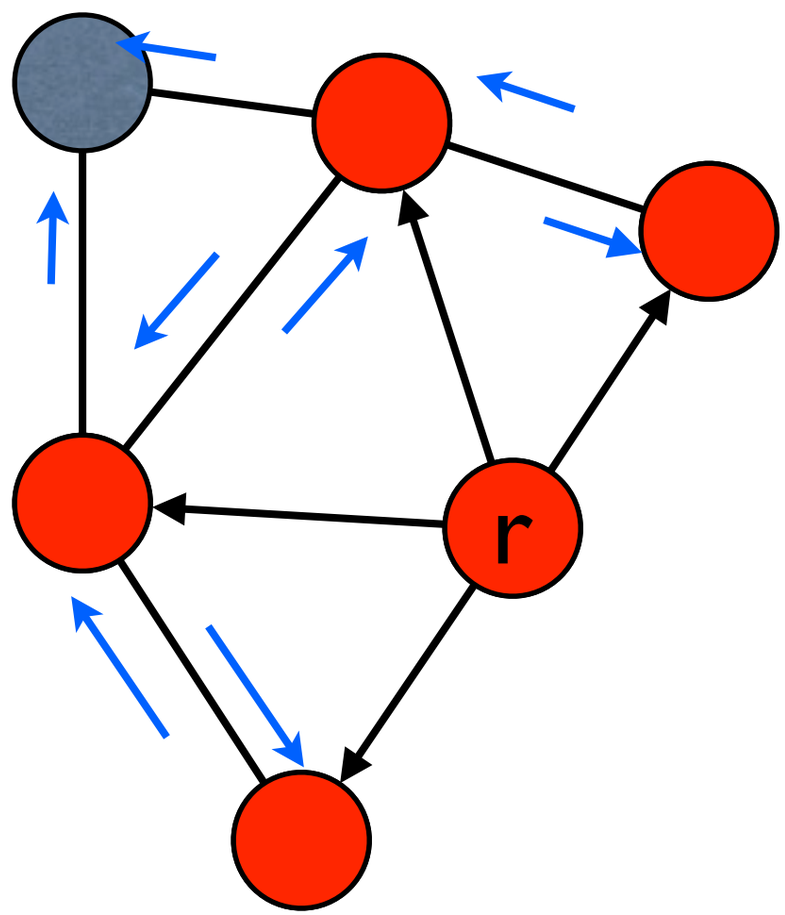

Dijkstra-like BFS

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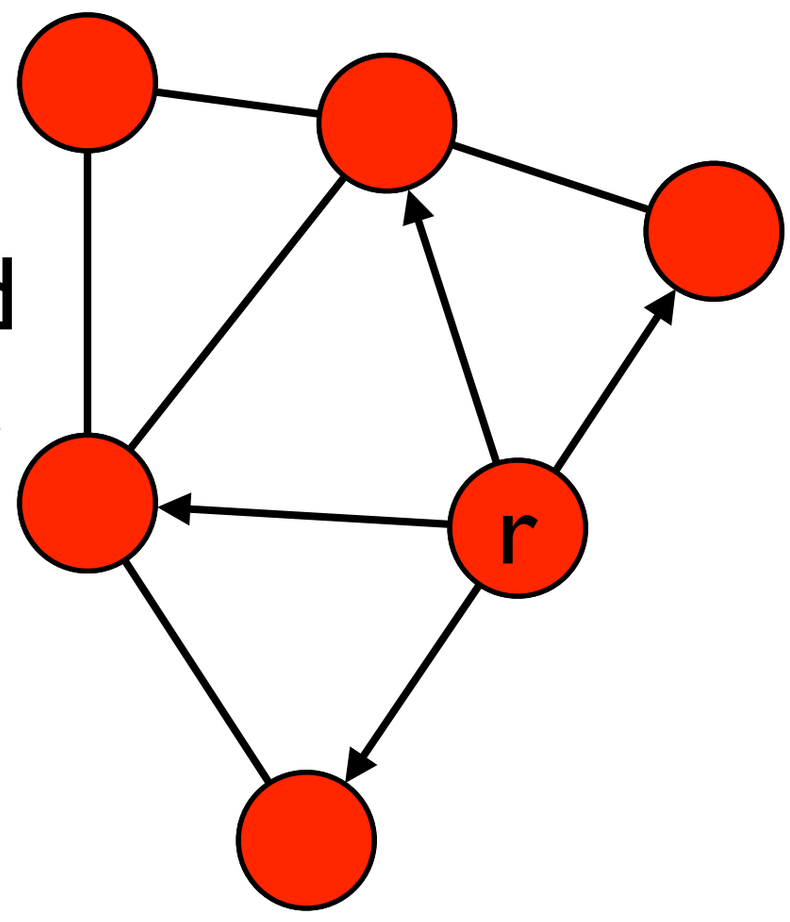

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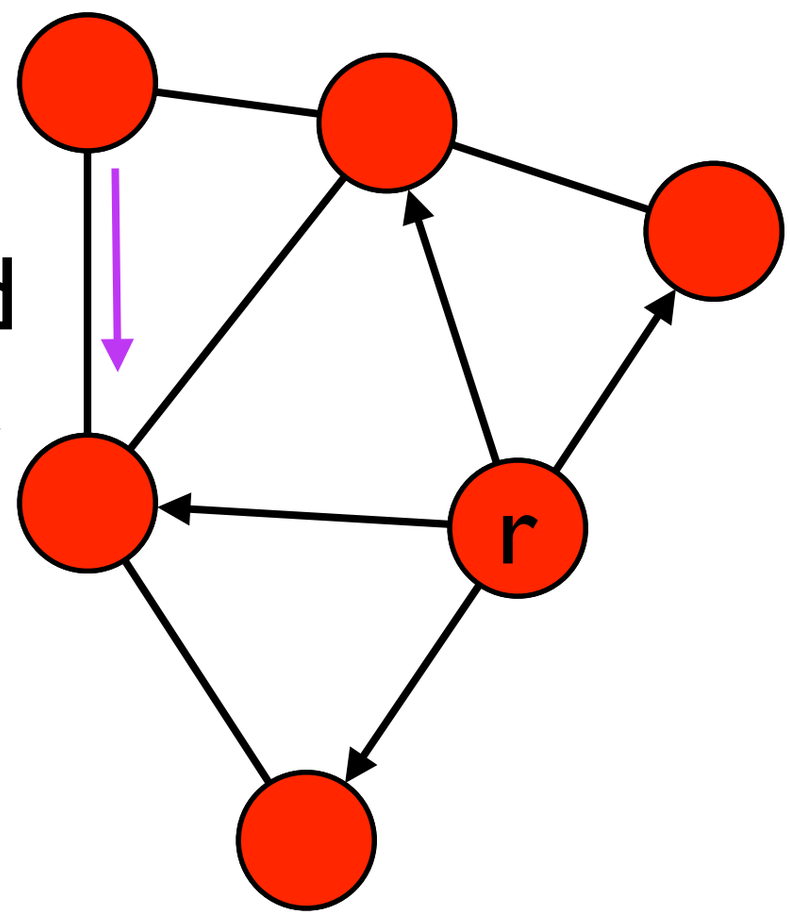

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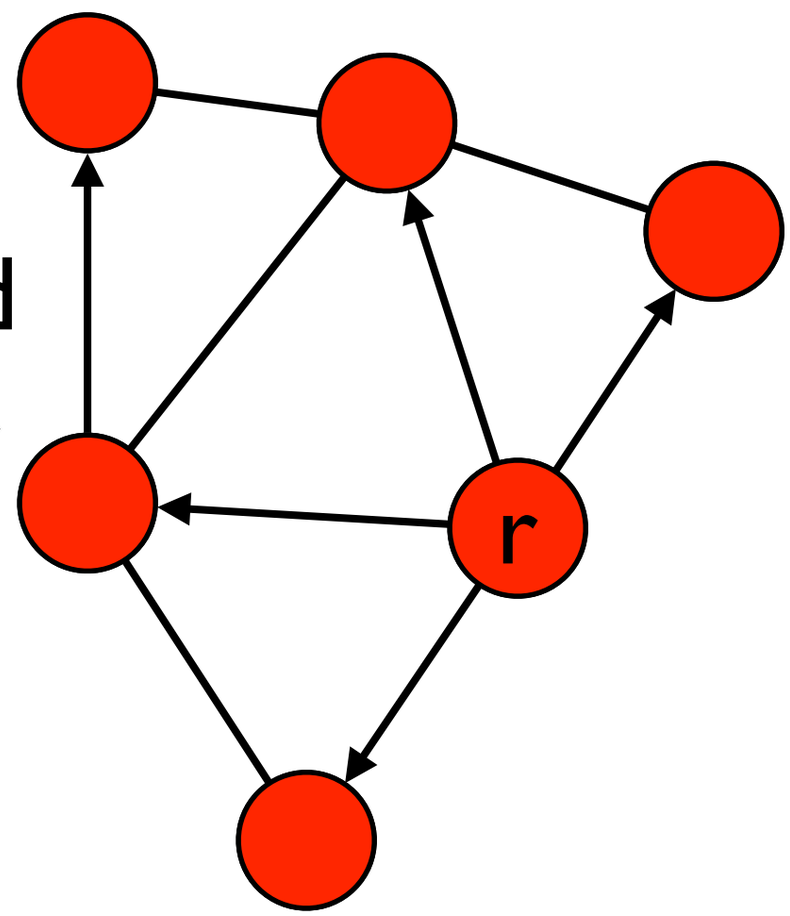

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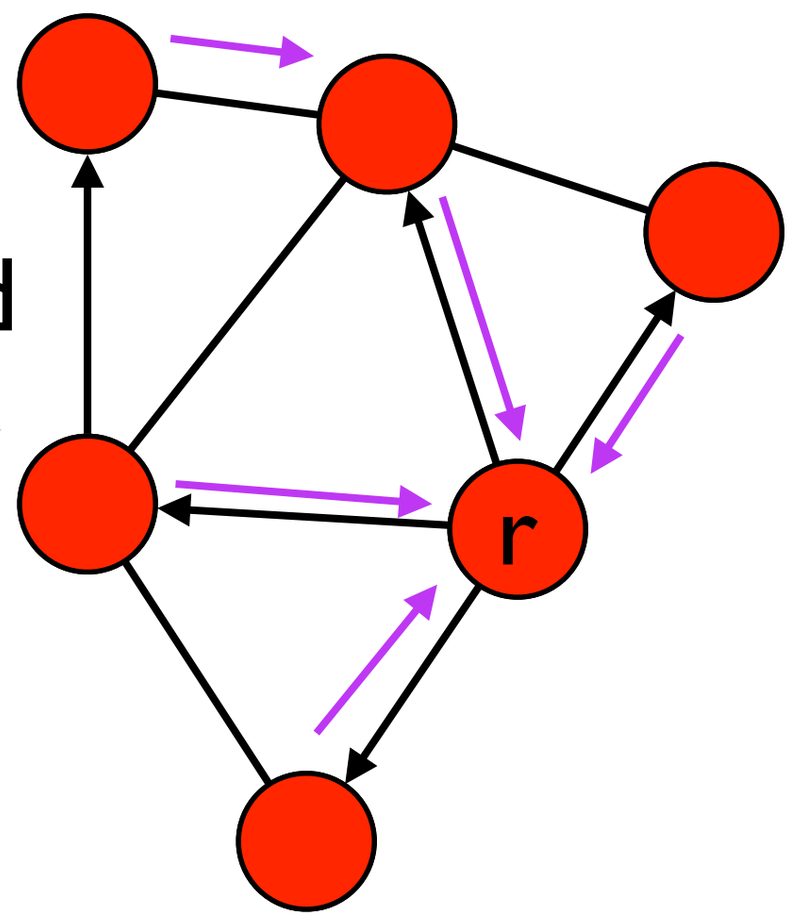
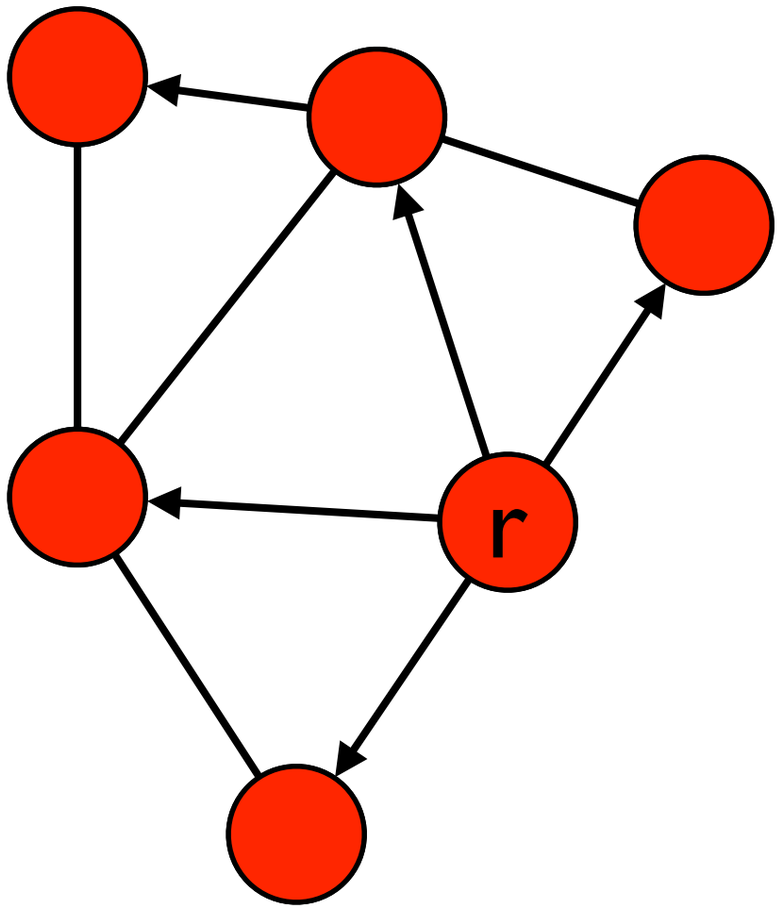


Dijkstra-like BFS ?



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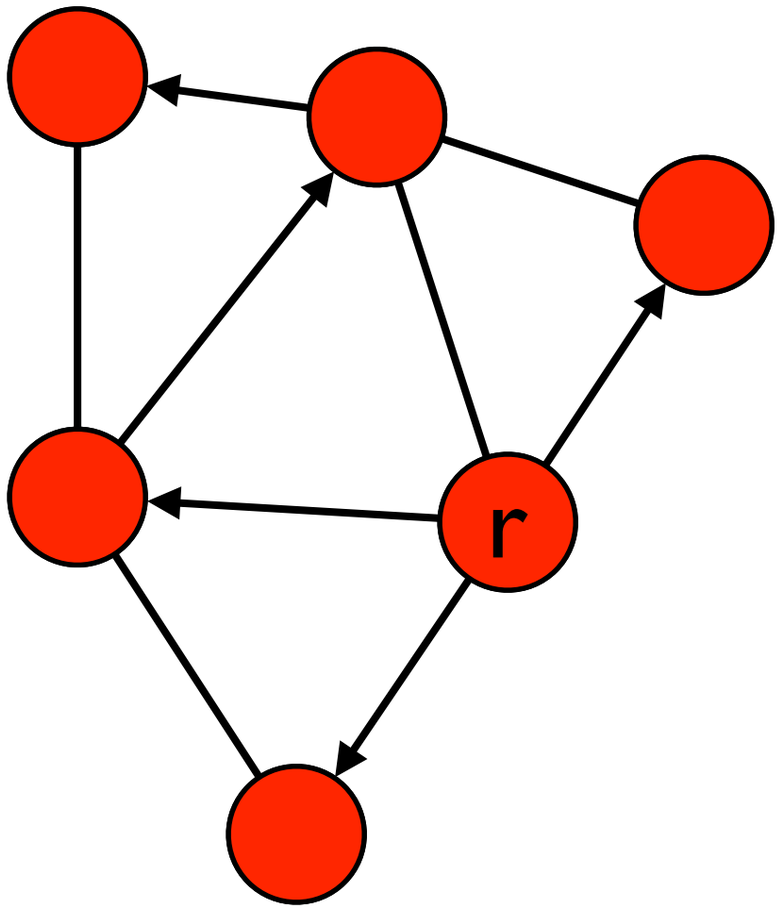


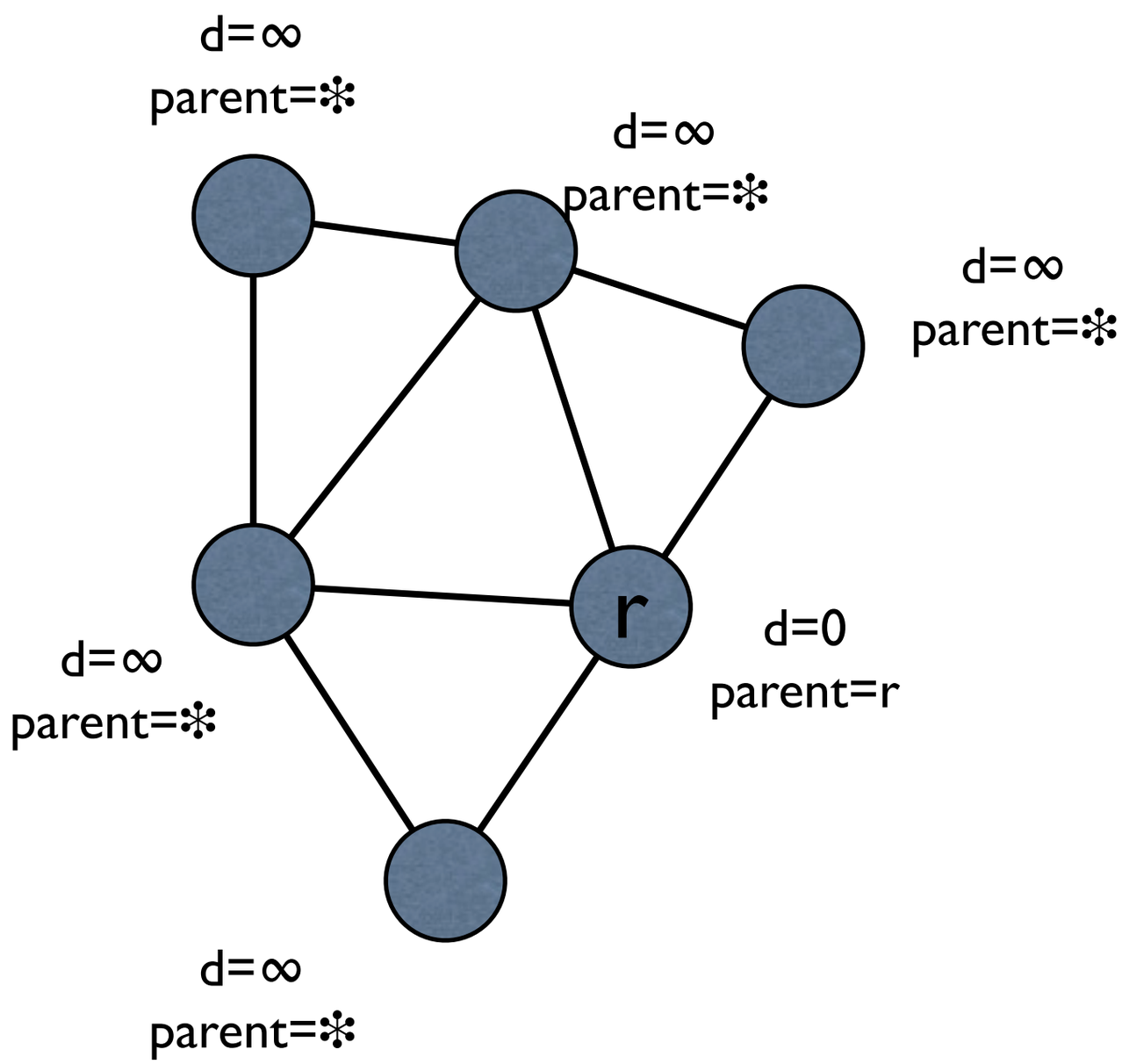
Dijkstra-like BFS - Complexity

- Time Complexity: $O(Diam^2(G))$
- Message Complexity: $O(nDiam(G)+|E|)$

Bellman-Ford-like BFS

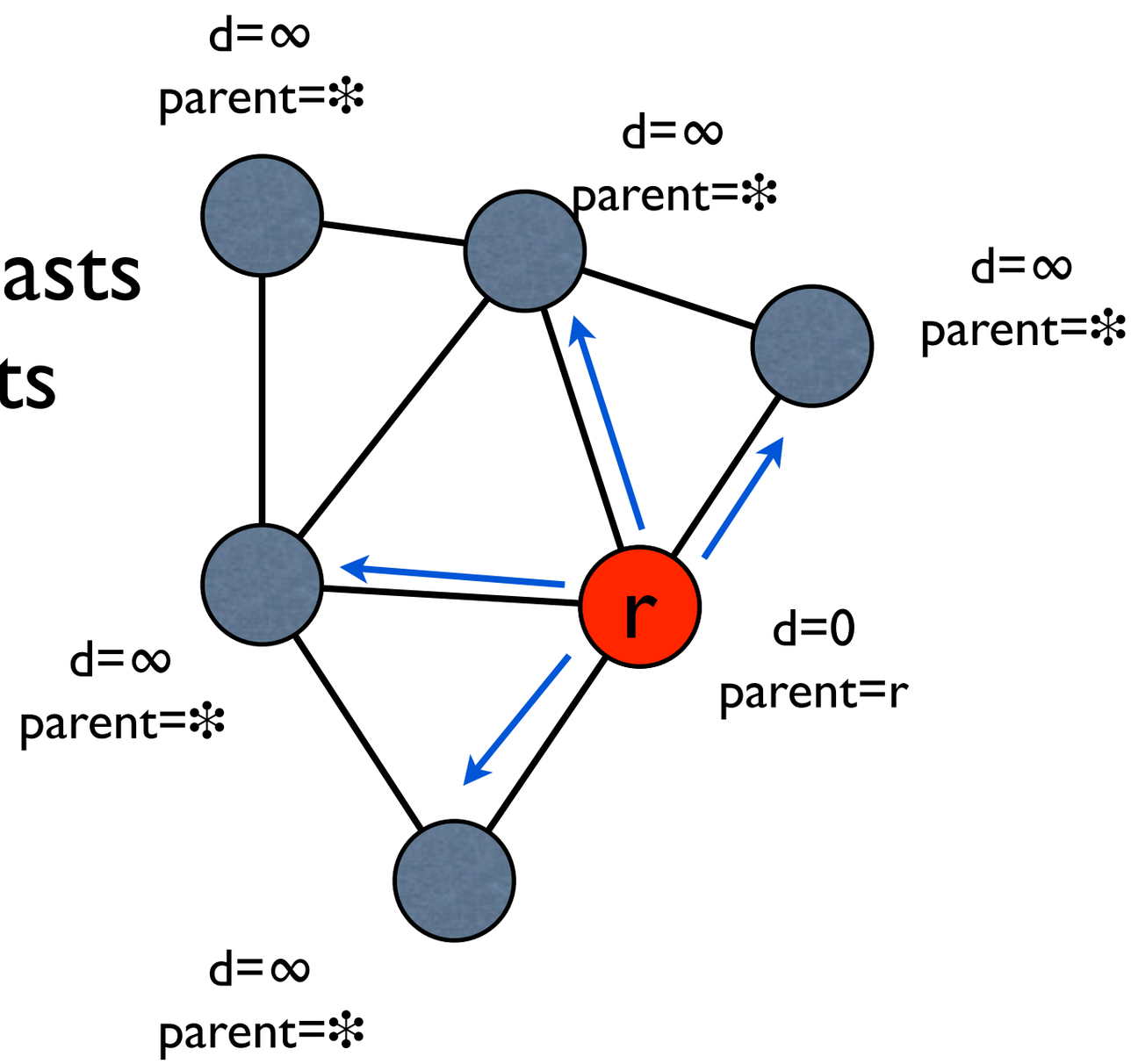
- each node maintains :
  - its shortes distance to the root d (initially d=infinity)
  - its parent to the root on the shortest distance

Bellman-Ford-like BFS



Bellman-Ford-like BFS

root, r, broadcasts d=0 to all its neighbors



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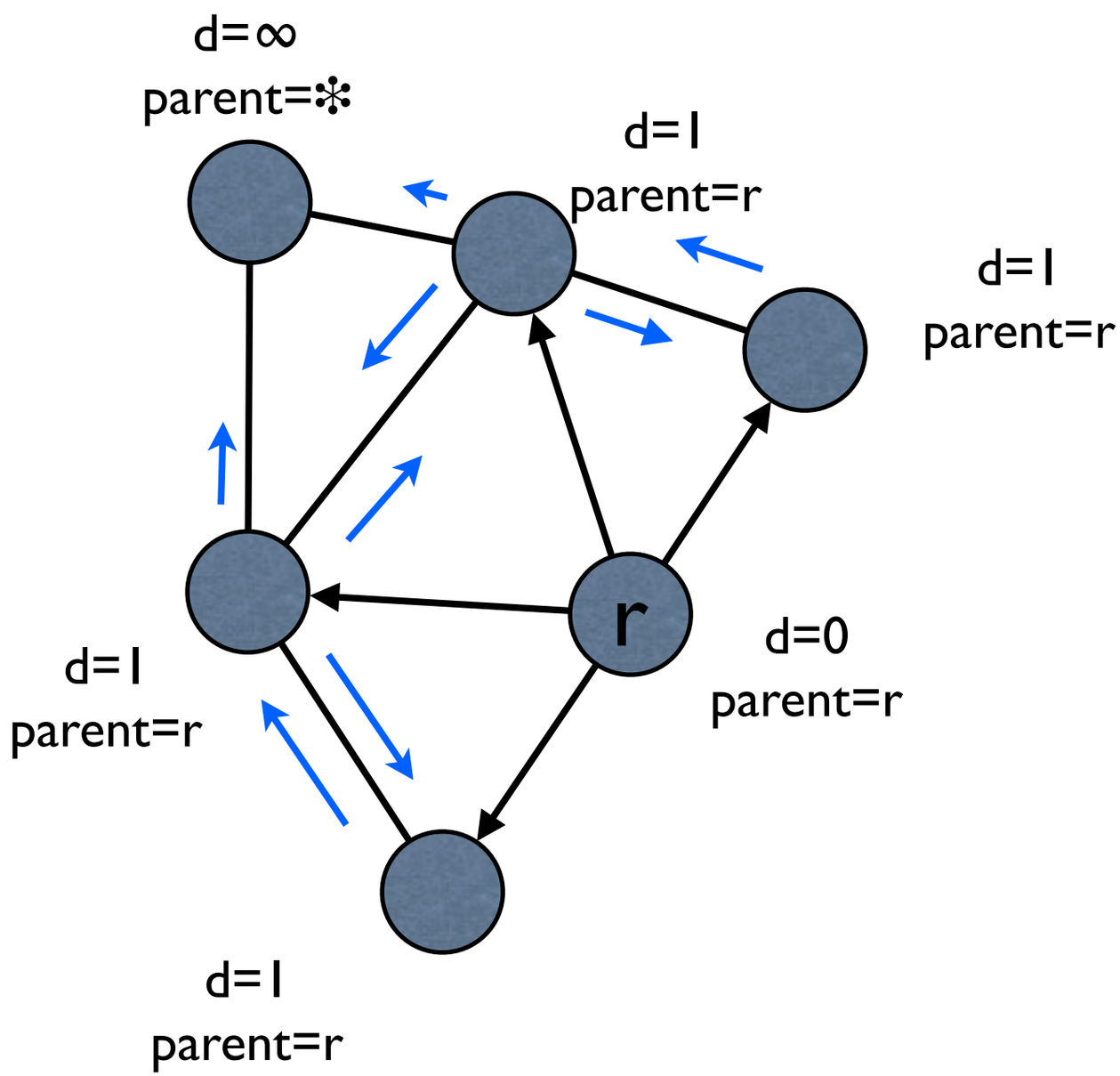
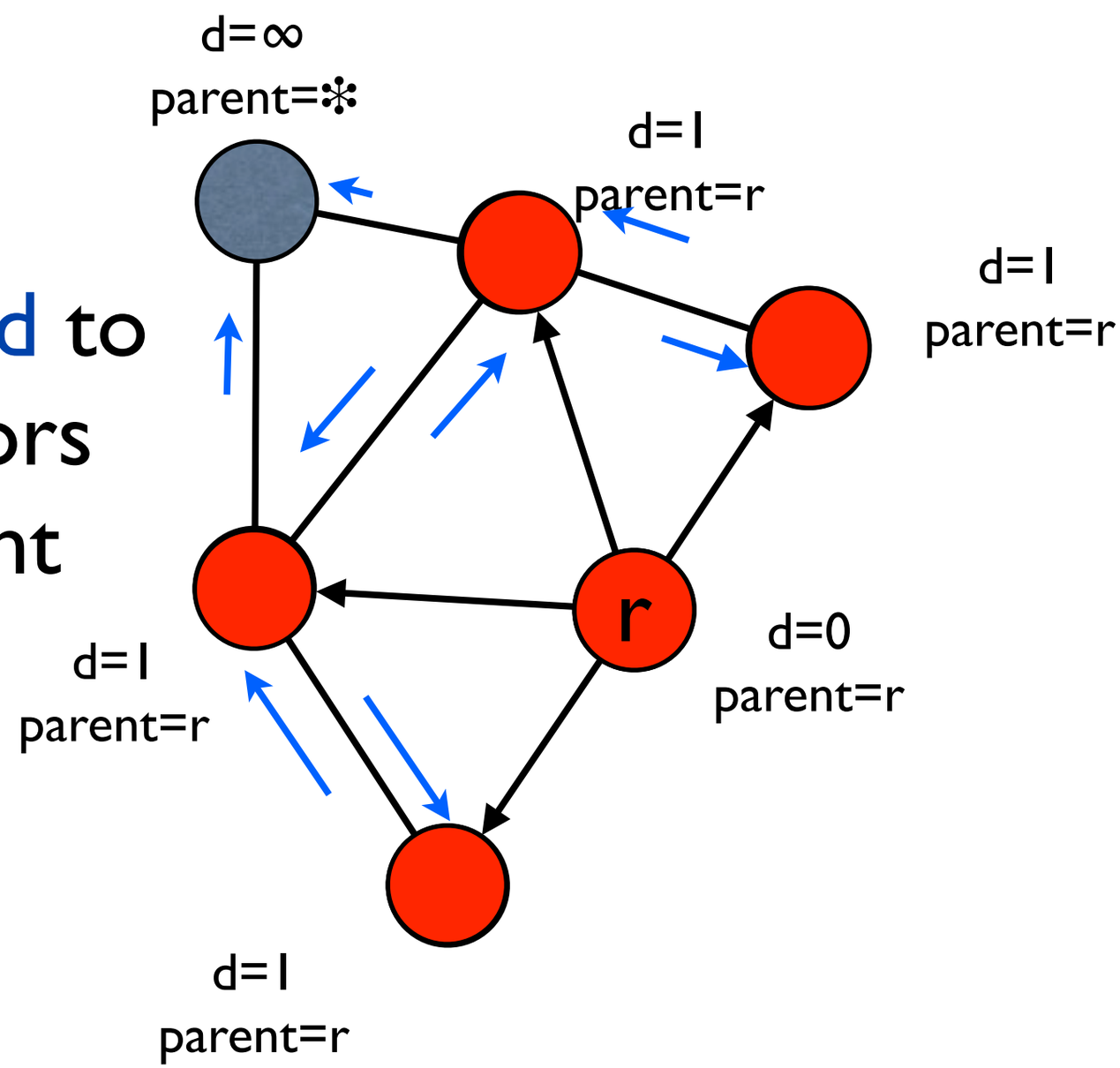


Bellman-Ford-like BFS

each node broadcasts its d to all its neighbors except parent



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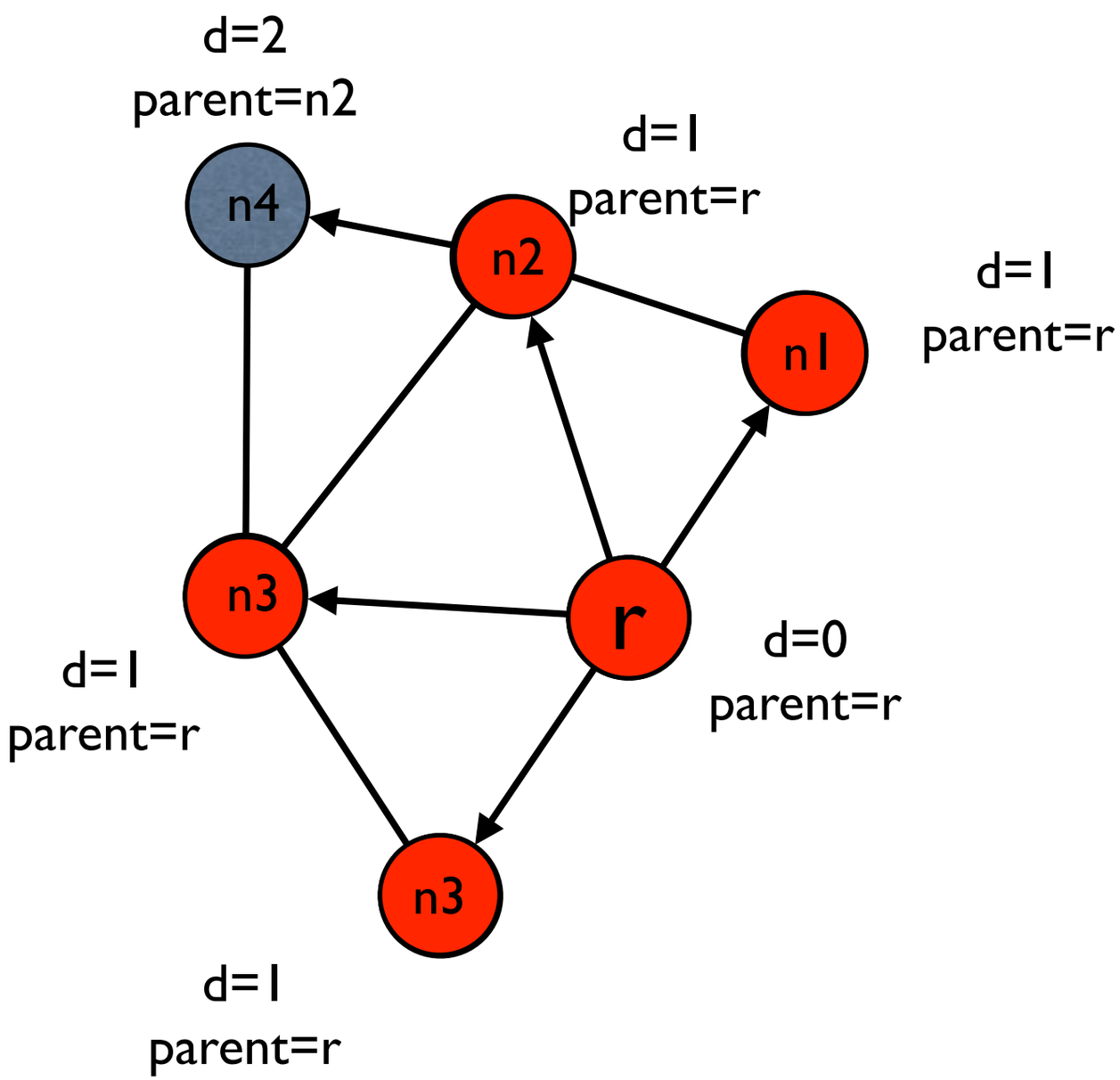

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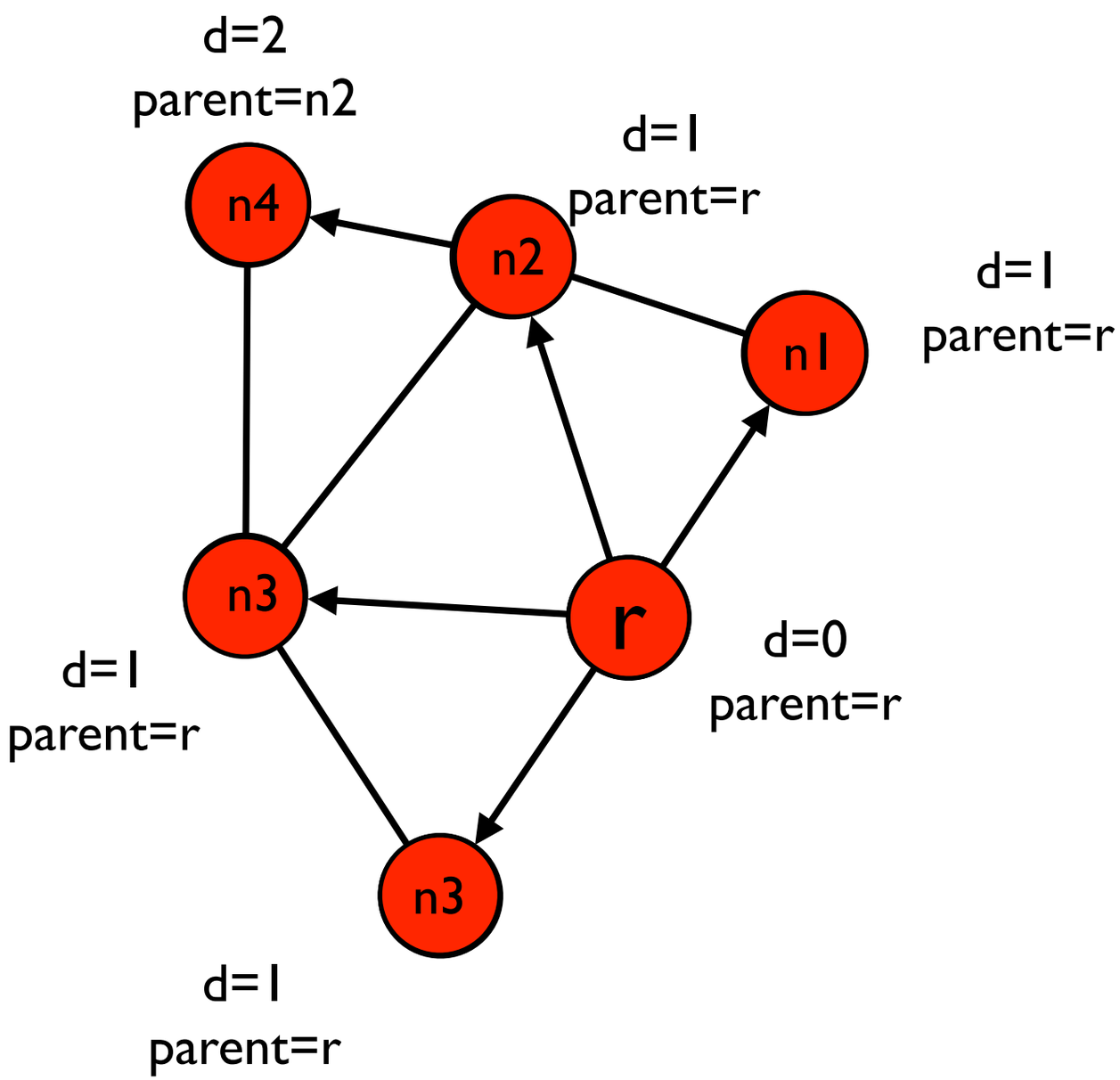

Bellman-Ford-like BFS - Complexity

- Time Complexity: O(Diam(G))
- Message Complexity: O(|E|)

BFS - Complexity

	Time	Messages
Dijkstra-like	D^2	$E+nD$
Bellman-Ford-like	D	nE
Awebuch-Peleg	$D\log^3 n$	$E+n\log^3 n$
Lower Bound	D	E

Spanning Tree and Routing

Routing

- «link state» (Dijkstra like)
  - Open Shortest Path First (OSPF)

- «distance vector» (Bellman-Ford like)
  - Border Gateway Protocol (BGP)
  - Interior Gateway Routing Protocol (IGRP, CISCO property)