

## Proof

- Begining: things we assume to be true, including the definitions of the things we talk about
- Middle: statements, each following
logically from the things before it
- End: the thing we're trying to prove

How to Write a Proof

- Managing Experimental Data
- Classical vs. Exploratory
- Practicalities


## Agenda



How to write proofs: a quick guide. Eugenia Cheng. http://www.math.uchcago.edu/~eugenia

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Example 1. Using the field axioms, prove that \(a(b-c)=a b-a c\) for any real numbers \(a, b, c\). You may use the fact that \(x .0=0\) for any real number \(x\).
Beginning field axioms
        definition x-y =x+(-y)
        given }x.0=



\section*{What is Wrong ?}
\[
\begin{aligned}
a(b-c) & =a b-a c \\
a b+a(-c) & =a b-a c \\
a(-c) & =-a c \\
a c+a(-c) & =0 \\
a(c+(-c)) & =0 \\
a .0 & =0 \\
0 & =0
\end{aligned}
\]

\section*{What is Wrong ?}
\(a(b-c)=a b+a(-c)\)
\(a(-c)=-a c \quad\) because if you add ac to
both sides then both sides vanish which means they're inverse
If result is true for \(\mathrm{n}=\mathrm{k}\) then
\(1+\cdots+k+(k+1)=\frac{k(k+1)}{2}+(k+1)\)
\(\square\)
Beginving Principle of Induction
MIDDLE \(\quad\) for \(\mathrm{n}=1\), LHS \(=\)
RHS \(=\frac{1(1+1)}{2}\)
\(=1\)
\(={ }_{\text {result is }} \quad 1\) true for \(\mathrm{n}=1\)
\(1+\cdots+k+(k+1)=\frac{k(k+1)}{2}+(k+1)\)
\(=\frac{k(k+1)+2(k+1)}{2}\)
\(=\frac{(k+1)(k+2)}{2}\)
result true for \(\mathrm{k} \Rightarrow\) result true for \(\mathrm{k}+\)
\(\therefore\) by the Principle of Induction, the result is true for all \(\mathrm{n} \in \mathbb{N}\) \(\qquad\)

Traps and Pitfalls

\section*{What is Wrong ?}
\[
\begin{aligned}
a(b-c) & =a b+a(-c) \\
& =a b+a(-c)+a .1 \\
& =a b+a(1-c) \\
& =a b-a c
\end{aligned}
\]

\section*{Additional Pitfalls}
- Incorrect assumptions
- Incorrect use of definitions, or use of incorrect definitions
\[
\begin{aligned}
& \mathrm{f}(\mathrm{a})=\mathrm{f}\left(\mathrm{a}^{\prime}\right) \quad \Longrightarrow \mathrm{a}=\mathrm{a}^{\prime} \\
& \mathrm{g}(\mathrm{a})=\mathrm{g}\left(\mathrm{a}^{\prime}\right) \Longrightarrow \mathrm{a}=\mathrm{a}^{\prime}
\end{aligned}
\]
\[
\begin{aligned}
(g \circ f)(a)=(g \circ f)\left(a^{\prime}\right) & \Longrightarrow g(a) \circ f(a)=g\left(a^{\prime}\right) \circ f\left(a^{\prime}\right) \\
& \Longrightarrow a=a^{\prime}
\end{aligned}
\]
\[
\mathrm{g} \circ \mathrm{f} \text { is injective. } \quad \square
\]

\section*{Assumptions}
- You need to justify everything enough for your peers to understand it
- If in doubt, justify things more rather than less

\section*{\(x\) is purple}
" \(x\) is purple" means \(y\)

We know
a and
\(\begin{aligned} \mathrm{a} & \Longrightarrow \mathrm{b} \\ & \Longrightarrow \mathrm{c} \\ & \Longrightarrow \mathrm{d} \\ & \Longrightarrow \mathrm{y}\end{aligned}\)
\(\therefore \quad \mathrm{x}\) is purple as required

\section*{Practicalities}
- Write the begining very carefully
- Write the end very carefully
- Try and manipulate both ends to meet in the middle, from big leaps to smaller ones
- Pretend to be more stupid (or sceptical, or untrusting) that you are

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\section*{\(\forall x, p(x)\) is true}

Prove that any rational number can be expressed as \(\frac{m}{n}\) where \(m\) and \(n\) are integers that are not both even.

Let x be a rational number. So x can be expressed as \(\frac{\mathrm{p}}{\mathrm{q}}\) where p and q are integers and \(\mathrm{q} \neq 0\).

\(\exists x\) s.t. \(p(x)\) is true
\(\exists \delta>0\) s.t. \(|x|<\delta \Longrightarrow\left|x^{2}\right|<\frac{1}{100}\)

Put \(\delta=\frac{1}{10}\). Now \(\left|\mathrm{x}^{2}\right|=|\mathrm{x}|^{2}\) so we have
\[
|x|<\frac{1}{10} \Longrightarrow\left|x^{2}\right|<\frac{1}{100}
\]

If \(a, b, c, d\) are true then \(e\) is true


\section*{Proof by Contradiction}
- We are trying to prove that some statement \(P\) is true
- We say «suppose \(P\) were not true» and find a contradiction
- Since \(P\) being false gives a contradiction, we deduce that \(P\) must be true

\section*{Graphical techniques}
- Plotting the raw data (data traces, histograms, bihistograms, probability plots, lag plots, block plots, and Youden plots)
- Plotting simple statistics such as mean plots, standard deviation plots, box plots, and main effect plots of the raw data
- Positioning such plots so as to maximize our natural pattern recognition abilities

\section*{Exploratory Data}

Analysis
I. Problem
2. Data
3. Analysis
4. Model
5. Conclusion


\section*{Model}
- Exploratory
- does not impose deterministic or probabilistic models on the data. In fact, EDA allows the data to suggest admissible models that best fit the data.

\section*{Techniques}

\section*{- Classical}
- Quantitative. Mean, Variance, ANOVA,Ttest, chi^ \({ }^{2}\) tests, F -Test.

\section*{- Exploratory}
- Graphical. Scatter plots, Character plots, box plots, histograms, bihistograms, probability plots, residual plots, mean plots.

\section*{Assumptions}

\section*{- Classical}
- Tests based on classical techniques are very sensitive. Yet they depend on underlying assumptions. that could be unkown or untested.

\section*{- Exploratory}
- Makes no assumptions.

\section*{Quantitative Techniques}
- Hypothesis testing
- Analysis of variance
- Point estimate and confidence intervals
- Least squares regression

\section*{Graphical Techniques}
- Testing assumptions
- Model Validation
- Estimator Selection
- Relationship identification
- Factor Effect determination
- Outlier Detection

\section*{EDA Example (DSI)}
- \(\mathrm{N}=1 \mathrm{l}\)
- Mean of \(X=9.0\)
- Mean ofY \(=7.5\)
- Intercept = 3
- Slope \(=0.5\)
- Residual Standard Deviation \(=1.237\)
- Correlation \(=0.816\)

\section*{EDA Example}
\begin{tabular}{|c|c|c|c|c|c|}
\hline x2 & Y2 & x3 & Y3 & X4 & Y4 \\
\hline 10.00 & 9.14 & 10.00 & 7.46 & 8.00 & 6.58 \\
\hline 8.00 & 8.14 & 8.00 & 6.77 & 8.00 & 5.76 \\
\hline 13.00 & 8.74 & 13.00 & 12.74 & 8.00 & 7.71 \\
\hline 9.00 & 8.77 & 9.00 & 7.11 & 8.00 & 8.84 \\
\hline 11.00 & 9.26 & 11.00 & 7.81 & 8.00 & 8.47 \\
\hline 14.00 & 8.10 & 14.00 & 8.84 & 8.00 & 7.04 \\
\hline 6.00 & 6.13 & 6.00 & 6.08 & 8.00 & 5.25 \\
\hline 4.00 & 3.10 & 4.00 & 5.39 & 19.00 & 12.50 \\
\hline 12.00 & 9.13 & 12.00 & 8.15 & 8.00 & 5.56 \\
\hline 7.00 & 7.26 & 7.00 & 6.42 & 8.00 & 7.91 \\
\hline 5.00 & 4.74 & 5.00 & 5.73 & 8.00 & 6.89 \\
\hline
\end{tabular}

\section*{EDA Example}

\section*{EDA Example}
\begin{tabular}{rr}
\multicolumn{1}{c}{X} & Y \\
10.00 & 8.04 \\
8.00 & 6.95 \\
13.00 & 7.58 \\
9.00 & 8.81 \\
11.00 & 8.33 \\
14.00 & 9.96 \\
6.00 & 7.24 \\
4.00 & 4.26 \\
12.00 & 10.84 \\
7.00 & 4.82 \\
5.00 & 5.68
\end{tabular}

EDA Example



\section*{EDA Example (DS3)}
- \(N=11\)
- Mean of \(X=9.0\)
- Mean of \(Y=7.5\)
- Intercept \(=3\)
- Slope \(=0.5\)
- Residual Standard Deviation \(=1.236\)
- Correlation \(=0.816\)

\section*{EDA Example (DS4)}
- \(N=11\)
- Mean of \(X=9.0\)
- Mean of \(Y=7.5\)
- Intercept = 3
- Slope \(=0.5\)
- Residual Standard Deviation \(=1.236\)
- Correlation \(=0.817\)


\section*{Data Sets}
- Flow DS: This data set was collected by Bob Zarr of NIST in January 1990 from a heat flow meter calibration and stability analysis. The response variable is a calibration factor.

EDA Example (DS2)


\section*{Four Basic Tools}

\section*{Data Sets}
- Walk DS:A random walk can be generated from a set of uniform random numbers by the formula
\[
R_{i}=\sum_{j=1}^{i}\left(U_{j}-0.5\right)
\]
- where \(U\) is a set of uniform random numbers

\section*{Univariate Data}

- Most basic tools operate on univariate data, i.e. a list of single responses

Beam DS:This data set was collected by H.S. Lew of NIST in 1969 to measure steelconcrete deflections. The response variable is the deflection of a beam from center point.
\begin{tabular}{|c|}
\hline Run-sequence Plot \\
\hline \begin{tabular}{l}
- Considers Univariate Data \\
- Vertical axis: response variable \(Y(i)\) \\
- Horizontal Axis: Index \(i(i=1,2,3, \ldots\) )
\end{tabular} \\
\hline
\end{tabular}

\section*{Run-sequence Plot}
- Used to answer the questions
- Are there any shifts in location ?
- Are there any shifts in variation ?
- Are there any outliers ?

Run-sequence Flow DS


\section*{Lag Plot}
- Considers univariate data
- Vertical Axis: \(Y(i)\) for all \(i\)
- Horizontal Axis: \(Y(i-I)\) for all \(i\)

Lag Plot Walk DS



\section*{Beyond Histograms :} (Normal) Cumulative



\section*{Histogram}
- Used to answer the following questions
- What kind of population do the data come from ?
- Where are the data located ?
- How spread out are the data ?
- Are the data symmetric or skewed ?
- Are there outliers in the data ?
(Normal) Probability Plot
- Considers univariate data
- Vertical axis: Ordered Response values
- Horizontal axis: Normal order statistics median

\section*{(Normal) Probability} Plot
- Used to answer the following questions:
- Are the data normally distributed ?
- What is the nature of the departure from normality (data skewed, shorted than expected tail, longer than expected tails, etc.) ?
\(\qquad\)
(Normal) Probability Plot Flow DS


Investigating Relationships

Gnuplot in action: Understanding data with graphs. Philipp K. Janert. Manning.
http://www.manning.com/janert/

\section*{Example 2:}

The 1970 Draft Lotery

(Normal) Probability Plot Walk DS


\section*{Scatter plots}
- Assumes Bivariate data, i.e. lists of 2-tuples of responses
- The point is to check the nature of the relationship between the two responses
- Take care of outliers

\section*{Example 2:}

The 1970 Draft Lotery


Example I:Traffic Pattern at Website


\section*{The Core Principle}
- Plot exactly what you want to see

Iterate \& Transform




Truncation \&
Responsiveness
- Outlier removal
- Sampling bias
- Edge effects

Improving Perception:
Banking


Normalized Metrics


Truncation \&
Responsiveness

Improving Perception


\section*{Improving Perception:}

Banking


Enhancing Quantitative
Perception
\(\square\)



\section*{GNUPLOT}
- Free software for plotting data
- NOT «push-button-limited-capacities» type of software
- Multiplatform
- Integrates well with LaTeX

Enhancing Quantitative
Perception


\section*{The Core Principle}
- Plot exactly what you want to see

\section*{GNUPLOT Invocation}
```

O
cic
M
Coprright (C) 1986-1993, 1998, 2004, 2007-2009
M
The gnuplot ERQ is avai, abel from

```

```

$$
\begin{subarray}{c}{\mathrm{ Terminal type set to 'x1I'}}\\{\mathrm{ gnuplot>}}\end{subarray}
$$

```


\section*{Plotting From Data}


\section*{Data Transformation}
plot "data" using 1:( sqrt(\$2) ) with lines plot "data" using 1:( (\$2+\$3)/2 ) with lines plot "data" using 1:2 with lines, " " using 1:( \(\$ 3 / 100\) ) with lines plot "data" using ( \(\log (\$ 1)):(\log (\$ 2)\) ) with lines

> set logscale
> set logscale \(x\) set logscale y
> unset logscale unset logscale x unset logscale y

\section*{plot \(\sin (x), x, x-(x * * 3) / 6\) \\ }
plot [] [-2:2] \(\sin (x), x, x-(x * * 3) / 6\)

plot "prices" using 1:2 title "PQR" with lines, \(\Rightarrow\) "prices" using 1:3 title "XYZ" with linespoint

set datafile separator ': '
set datafile commentschar "m"
plot t-20.150)
- u 3:(\$0+2):( stringcolumn(1) . "\n" . stringcolumn(5)) w labels


Run-sequence Flow DS


\section*{Exporting Graphics}
- «Web» graphics
- JPG, SVG, PNG, GIF
- «Print» graphics
- EPS, EPSLaTeX, PDF

\section*{Implementing EDA 4BT}
- Run-sequence Plot
- Lag Plot
- Histogram
- (Normal) Probability Plot

\section*{Exporting EPS}
\# plot commands
set terminal postscript eps enhanced set output 'enhanced.eps'
replot

\section*{Including EPS in LaTeX}

-

\section*{Including EPS in LaTeX}
\documentclass\{article\}
Kusepackage \{graphicx\}
\begin\{document\} }
section\{The First Section\}
\(\backslash\) begin \(\{\) figure \(\}\) [h]
lbegin \{center\}
includegraphics [width \(=10 \mathrm{~cm}\) ] \{enhanced\}
\caption\{A Postscript file, included in \LaTeX\}
\end\{figure\} }
\end\{document\} }
\[
=
\]

\section*{Run-sequence Plot}
set terminal postscript eps color "Times-Roman" 16
set output "flowmeter_runseq.eps"
plot "flowmeter1" with lines



(Normal) Probability Plot
Set output
"flowmeter_isnormal.eps" plot "flowmeter_cdf" using (invnorm(\$2)):1 with lines


\section*{(Normal) Probability} Plot
set output "flowmeter_cumulative.eps" plot "flowmeter1" usīng 1:(1./195.) plot 1 owmeter \({ }^{\prime \prime}\)


(Normal) Probability Plot Flow DS


Histogram
set output
"flowmeter_histogram.eps"
bin ( \(x, s\) ) =-s*int (x/s)
set boxwidth 0.01
plot "flowmeter1" using
(bin (\$1,0.01)): (1./(0.01*195))
flowmeter I
smooth frequency with boxes


\section*{(Normal) Probability Plot}
```

set t
"flowmeter_cdf"
replot

```
    unset table
flowmeterl
flowmeter cdf


\section*{Deadline: \\ Tuesday 6 Nov., 17:00}
- Step I: Say something meaningful about the data located at the METHO class web site:
- Format:
- node \(x\), node \(y\), begin contact, end contact
- One page limit, One graph limit

\section*{Deadline: \\ Tuesday 6 Nov., 17:00}
- Step 2: download some experimental data:
- http://sensorscope.epf1.ch/index. ohp/Envi ionmental Data
- \(\frac{\mathrm{http} \text { ///fta. inria. } \mathrm{fr} / \text { apache } 2 \text {-default/pmui } \mathrm{ki} / \text { index. php? }}{\mathrm{n}=\mathrm{Ma} \text { in. Datasets }}\)

Can you confirm/invalidate associated publications? Can you extract new insight on the data ?

No page limit, no graph limit, include source code
Step I + Step 2 = One PDF```

