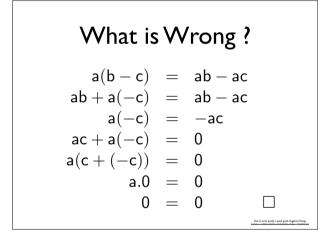


BEGINNING	definition of injective definition $(g \circ f)(a) = g(f(a))$		
	assumption that f and g are injectiv $\forall a, a' \in A f(a) = f(a') \Longrightarrow a =$ $\forall b, b' \in B g(b) = g(b') \Longrightarrow b =$	a′	
MIDDLE			by definition since g is injectiv since f is injectiv
	$\therefore (g\circ f)(a) = (g\circ f)(a') \implies$	a=a'	
END	i.e. g ∘ f is injective, as required		



Example 3. Prove by induction that
$$\forall n \in \mathbb{N}, 1 + \dots + n = \frac{n(n+1)}{2}$$

 BEGINNING Principle of Induction

 MIDDLE
 for n = 1, LHS = 1

 RHS = $\frac{1(1+1)}{2}$

 = 1

 \therefore result is true for n = 1

 If result is true for n = k then

 $1 + \dots + k + (k+1) = \frac{k(k+1)}{2} + (k+1)$

 = $\frac{k(k+1) + 2(k+1)}{2}$

 = $\frac{(k+1)(k+2)}{2}$

 i.e. result true for n = k + 1

 \therefore result true for k + 1

 END
 \therefore by the Principle of Induction, the result is true for all $n \in \mathbb{N}$

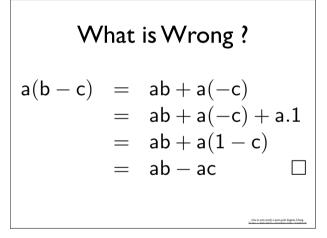
What is Wrong ? a(b-c) = ab + a(-c)= ab - ac

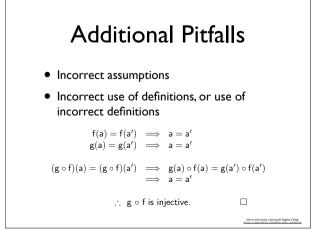
Beware Incorrect Logic

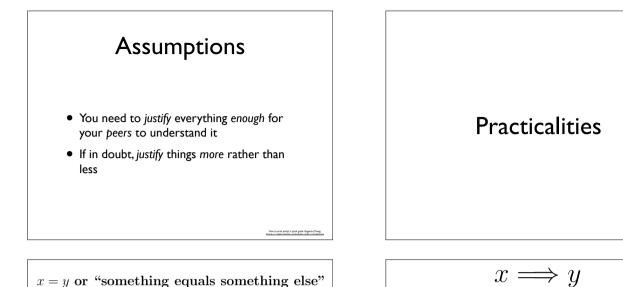
How to write proof: a quick guide. Eugenia Chang.

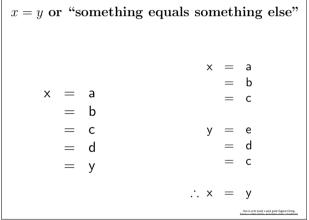
- Negating a statement incorrectly
- proving the converse of something instead of the thing itself

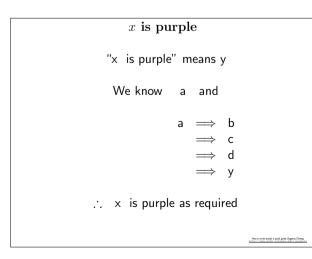
 $\begin{aligned} \forall \varepsilon > 0 \ \exists \delta > 0 \ \text{s.t.} \ \forall x \ \text{satisfying} \ 0 < |x - a| < \delta, \quad |f(x) - l| < \varepsilon \\ \exists \varepsilon > 0 \ \text{s.t.} \ \forall \delta > 0 \ \exists x \ \text{satisfying} \ 0 < |x - a| < \delta \ \text{s.t.} \ |f(x) - l| \geq \varepsilon \end{aligned}$







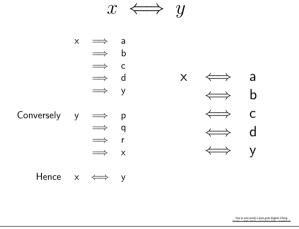




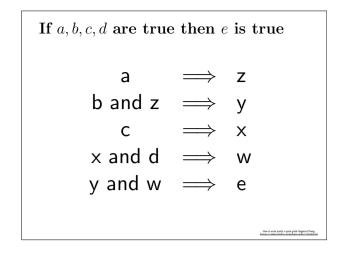
$$\forall x, p(x) \text{ is true}$$

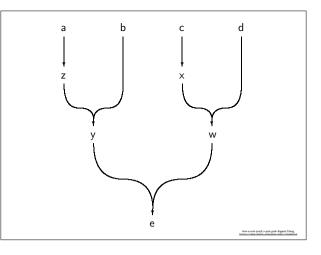
Prove that any rational number can be expressed as $\frac{m}{n}$ where m and n are integers that are not both even.
Let x be a rational number. So x can be expressed as $\frac{p}{q}$ where p and q are integers and $q \neq 0$.
...

Practicalities Write the begining very carefully Write the end very carefully Try and manipulate both ends to meet in the middle, from big leaps to smaller ones Pretend to be more stupid (or sceptical, or untrusting) that you are



$$\exists x \text{ s.t. } p(x) \text{ is true}$$
$$\exists \delta > 0 \text{ s.t. } |x| < \delta \implies |x^2| < \frac{1}{100}$$
$$\mathsf{Put } \delta = \frac{1}{10}. \text{ Now } |\mathsf{x}^2| = |\mathsf{x}|^2 \text{ so we have}$$
$$|\mathsf{x}| < \frac{1}{10} \implies |\mathsf{x}^2| < \frac{1}{100}$$





Approach

• Exploratory Data Analysis employs a

variety of (mostly graphical) techniques to:

• maximize insight into a data set

• uncover underlying structure

detect outliers and anomalies
test underlying assumptions

• develop parsimonious models

• determine optimal factor settings

• extract important variables

Exploratory Data Analysis

NIST/SEMATECH e-Handbook of Statistical Methods, http://www.itl.nist.gov/div898/handbook/

Classical vs. Exploratory



- We say «suppose P were not true» and find a contradiction
- Since P being false gives a contradiction, we deduce that P must be true

Graphical techniques

- Plotting the raw data (data traces, histograms, bihistograms, probability plots, lag plots, block plots, and Youden plots)
- Plotting simple statistics such as mean plots, standard deviation plots, box plots, and main effect plots of the raw data
- Positioning such plots so as to maximize our natural pattern recognition abilities

NSTSEMATICH = Handbook of Statistical Methods, http://www.itl.nist.gov/div#We/handboo

How to write proof: a quick guide. Eugenia Cheng http://www.math.uchcasco.edu/~eugeni

Classical Data Analysis

- I. Problem
- 2. Data
- 3. Model
- 4. Analysis
- 5. Conclusion

Exploratory Data Analysis

- I. Problem
- 2. Data
- 3. Analysis
- 4. Model
- 5. Conclusion

naturical Methods, d19999 /handbook /

Classical vs. Exploratory Models Focus Techniques Rigor Data Treatment Assumptions

Model

• Exploratory

 does not impose deterministic or probabilistic models on the data. In fact, EDA allows the data to suggest admissible models that best fit the data.

Focus

• Classical

• On the Model. Estimate model parameters, generate predicted values from the model.

• Exploratory

• On the Data. Structure, outliers, and models suggested by the data.

Techniques

- Classical
- Quantitative. Mean, Variance, ANOVA, Ttest, chi² tests, F-Test.
- Exploratory
- *Graphical*. Scatter plots, Character plots, box plots, histograms, bihistograms, probability plots, residual plots, mean plots.

NETSEMATICH + Handbook of Statistical Methods, https://www.iti.nist.cov/division/handbook/

Rigor

- Classical
- Probabilistic *foundation* of Science. Rigorous, formal, objective.
- Exploratory
- Suggestive, indicative, insightful. Subjective, depend on interpretation.

NETERATION - Randbook of Statistical Methods http://www.iti.nist.com/divisis/handbo

Data Treatment

• Classical

• Maps all data into *few* numbers. Loss of information.

• Exploratory

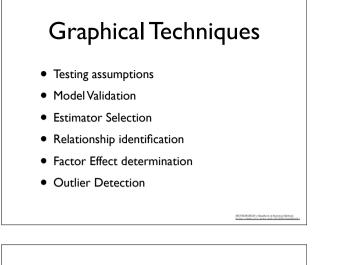
• Shows all data. No loss of information.

Assumptions

- Classical
 - Tests based on classical techniques are very sensitive. Yet they depend on underlying assumptions. that could be unkown or untested.
- Exploratory
- Makes no assumptions.

Quantitative Techniques

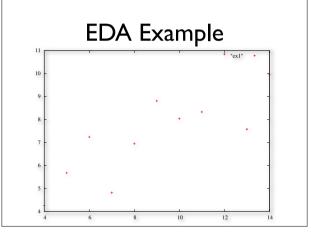
- Hypothesis testing
- Analysis of variance
- Point estimate and confidence intervals
- Least squares regression



EDA Example (DSI)

- N = 11
- Mean of X = 9.0
- Mean of Y = 7.5
- Intercept = 3
- Slope = 0.5
- Residual Standard Deviation = 1.237
- Correlation = 0.816

EDA Example						
X2	¥2	Х3	¥3	X4	¥4	
10.00	9.14	10.00	7.46	8.00	6.58	
8.00	8.14	8.00	6.77	8.00	5.76	
13.00	8.74	13.00	12.74	8.00	7.71	
9.00	8.77	9.00	7.11	8.00	8.84	
11.00	9.26	11.00	7.81	8.00	8.47	
14.00	8.10	14.00	8.84	8.00	7.04	
6.00	6.13	6.00	6.08	8.00	5.25	
4.00	3.10	4.00	5.39	19.00	12.50	
12.00	9.13	12.00	8.15	8.00	5.56	
7.00	7.26	7.00	6.42	8.00	7.91	
5.00	4.74	5.00	5.73	8.00	6.89	
				AISTISIA Istoi / An	TECH e-Handbard of Statistical Methods, eg. 153 - ni et nov/-di-vitit /handbaak/	

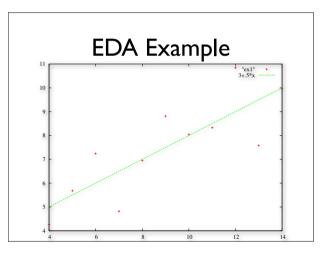


EDA Example

EDA Example (DS2)

- N = 11
- Mean of X = 9.0
- Mean of Y = 7.5
- Intercept = 3
- Slope = 0.5
- Residual Standard Deviation = 1.237
- Correlation = 0.816

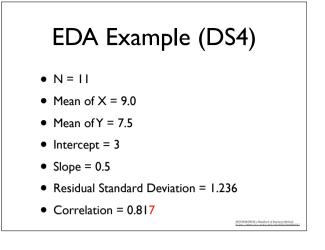
EDA Example				
Х	Y			
10.00	8.04			
8.00	6.95			
13.00	7.58			
9.00	8.81			
11.00	8.33			
14.00	9.96			
6.00	7.24			
4.00	4.26			
12.00	10.84			
7.00	4.82			
5.00	5.68 MITTER/PERIOD + Readers of Statistical Mitters/			

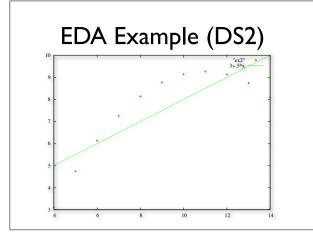


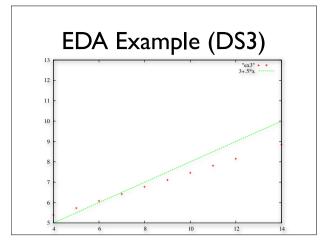
EDA Example (DS3)

- N = 11
- Mean of X = 9.0
- Mean of Y = 7.5
- Intercept = 3
- Slope = 0.5
- Residual Standard Deviation = 1.236
- Correlation = 0.816

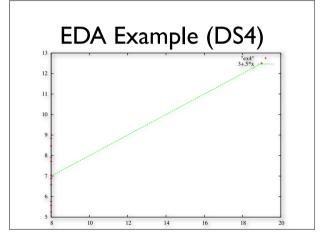
NISTISEMATECH + Handbook of Statistical B http://www.itl.mist.com/div#88/

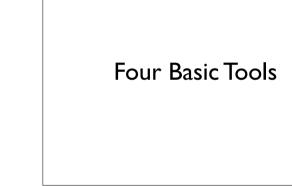


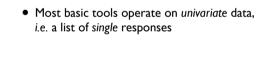




Univariate Data







Data Sets

• Flow DS: This data set was collected by Bob Zarr of NIST in January 1990 from a heat flow meter calibration and stability analysis. The response variable is a calibration factor. Data Sets

• Walk DS: A random walk can be generated from a set of uniform random numbers by the formula :

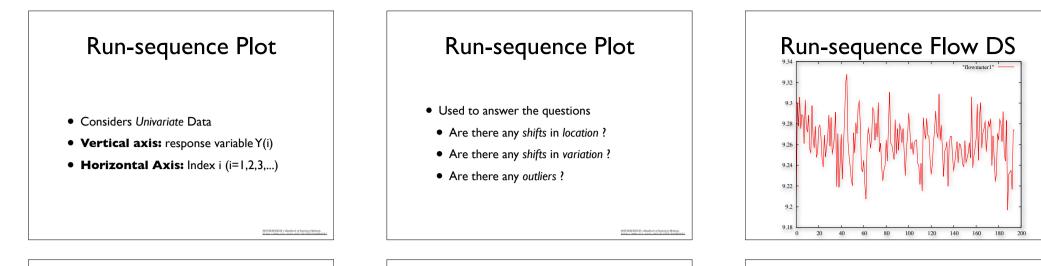
$$R_i = \sum_{j=1}^{i} (U_j - 0.5)$$

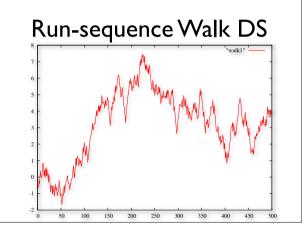
• where U is a set of uniform random numbers

Data Sets

• **Beam DS**: This data set was collected by H.S. Lew of NIST in 1969 to measure steelconcrete deflections. The response variable is the deflection of a beam from center point.

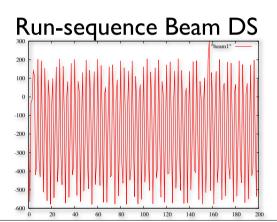
TSEMATECH e-Handbook of Statistical Methods, p://www.iti.nist.gov/div#98/handbook/





Lag Plot

- Are the data random ?
- Is there serial correlation in the data ?
- What is a suitable model for the data ?
- Are there outliers in the data ?



Lag Plot Flow DS

9.32 9.3

9.28

9.26

9.24

9.22 9.2 9.18 L 9.18

9.2

9.22

9.24

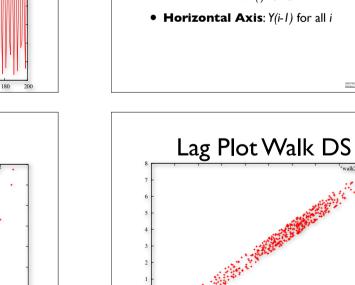
9.26

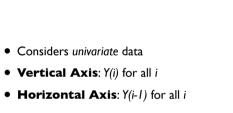
9.28

9.3

9.32

9.34



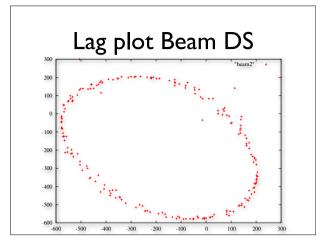


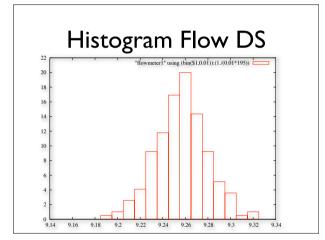
Lag Plot

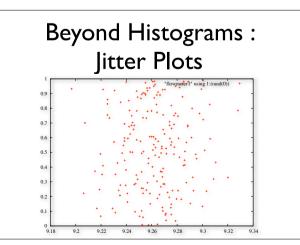


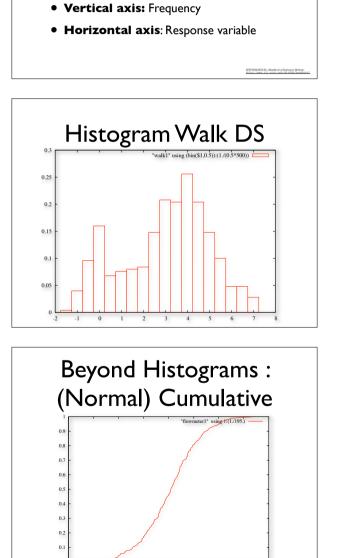
4











922 924 926 928 93

9.32

Histogram

• Split the range of the data into equal-sized

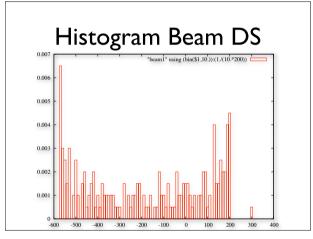
bins, then for each bin the number of points from the data for each bin are counted

• Considers univariate data

Histogram

- Used to answer the following questions
 - What kind of population do the data come from ?
 - Where are the data located ?
 - How spread out are the data ?
 - Are the data symmetric or skewed ?
- Are there outliers in the data ?

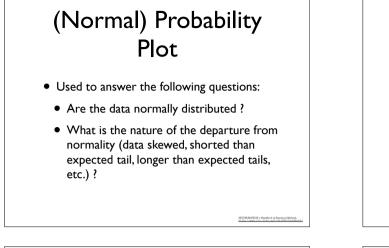
NSTSEMATECH + Handbook of Statistical Methods, http://www.itl.nist.gov/div898/handbook



(Normal) Probability Plot

- Considers univariate data
- Vertical axis: Ordered Response values
- Horizontal axis: Normal order statistics median

NEST-SEMATECH = Handbook of Statistical Methods, http://www.iti.nist.com/div#98/handb



_cdf" using (invnorm(\$2)):1

(Normal) Probability

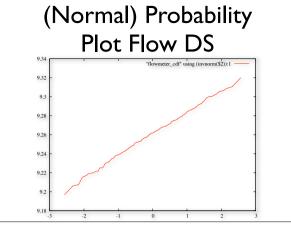
Plot Beam DS

· 200

-400

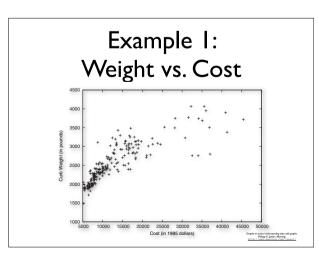
- 500

-600

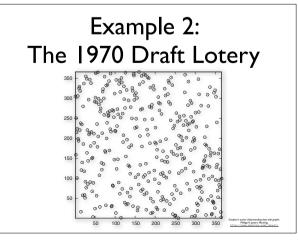


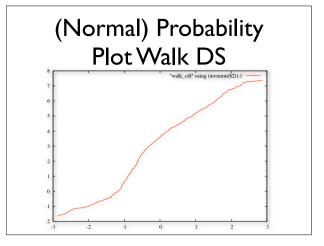
Investigating Relationships

Gnuplot in action: Understanding data with graphs. Philipp K. Janert. Manning. http://www.manning.com/janert/



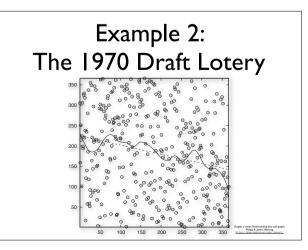
-1

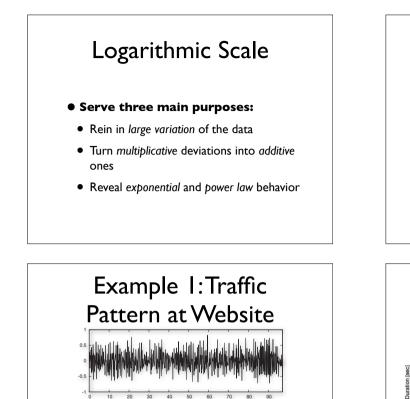


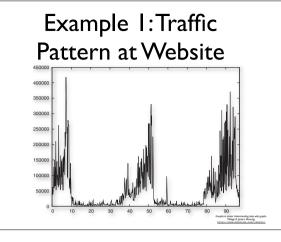


Scatter plots

- Assumes *Bivariate* data, i.e. lists of 2-tuples of responses
- The point is to check the nature of the *relationship* between the two responses
- Take care of *outliers*

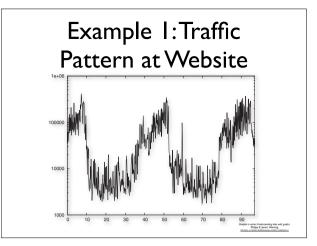


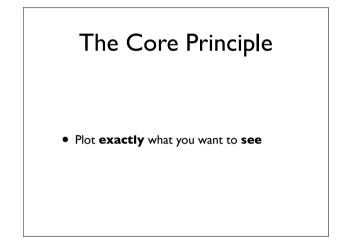


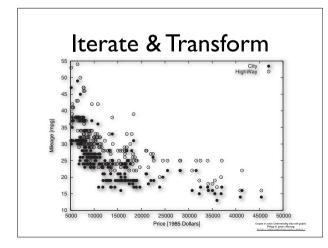


Example 2: Mammals

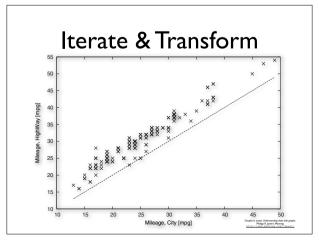
onlse







400k 350k



100

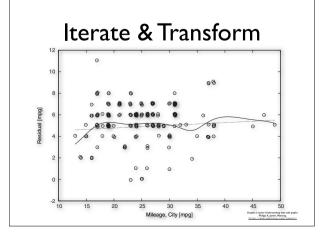
Weight [kg

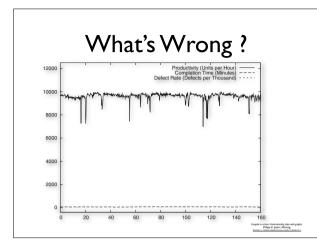
1000 10000

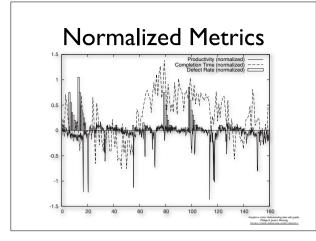
100000

1e+06

action Understanding data Philipp K. Janert. Monning



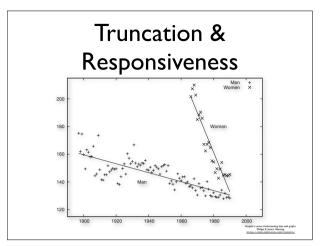


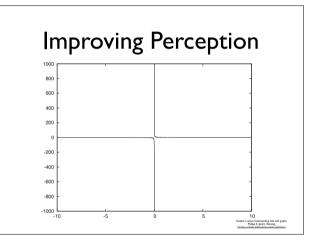


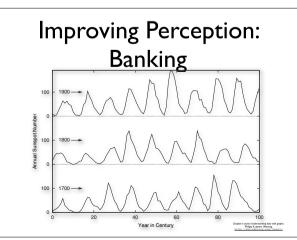
Truncating &

Responsiveness

Men + Vomen ×

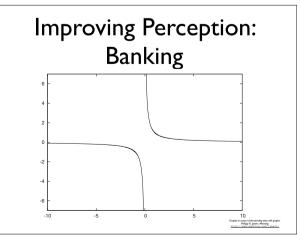


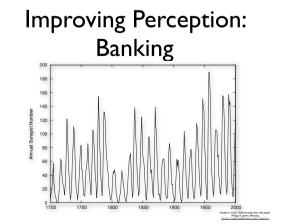


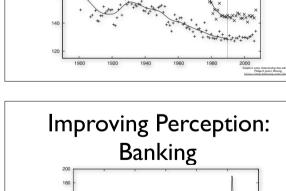


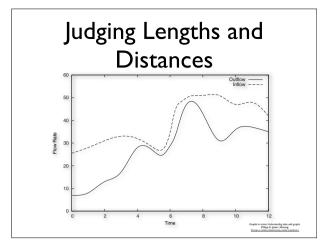


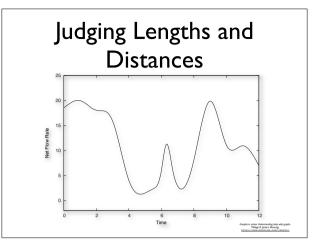
- Outlier removal
- Sampling bias
- Edge effects

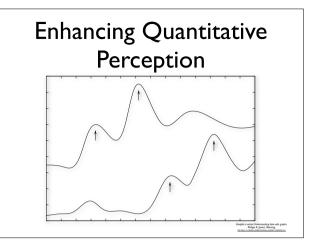


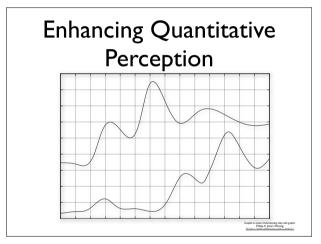


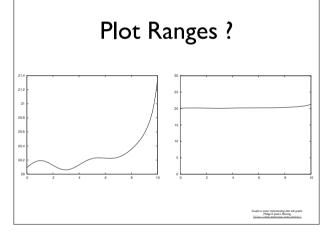


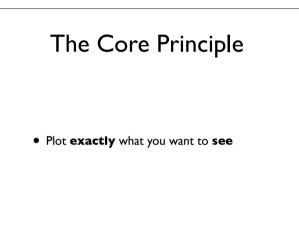














http://www.gnuplot.info

Gnuplot in action: Understanding data with graphs. Philipp K. Janert. Manning. http://www.manning.com/janert/

GNUPLOT

- **Free** software for plotting data
- NOT «push-button-limited-capacities» type of software
- Multiplatform
- Integrates well with *LaTeX*

GNUPLOT Invocation

Mac-Pro:metho tixeuil\$ gnuplot

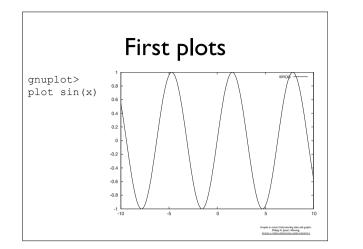
G N U P L O T Version 4.3 patchlevel 0 last modified March 2009 System: Darwin 9.8.0

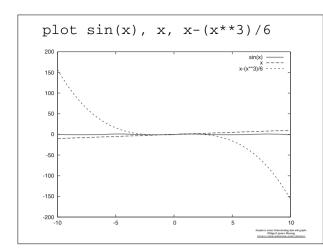
Copyright (C) 1986-1993, 1998, 2004, 2007-2009 Thomas Williams, Colin Kelley and many others

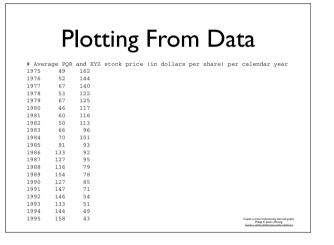
Type `help` to access the on-line reference manual. The gnuplot FAQ is available from <u>http://www.qnuplot.info/faq/</u>

Send comments and help requests to <<u>gnuplot-beta@lists.sourceforge.net</u>> Send bug reports and suggestions to <<u>gnuplot-beta@lists.sourceforge.net</u>>

Terminal type set to 'x11' gnuplot>





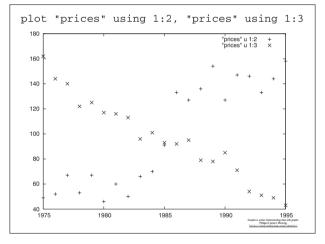


Data Transformation

plot "data" using 1:(sqrt(\$2)) with lines plot "data" using 1:((\$2+\$3)/2) with lines plot "data" using 1:2 with lines, "" using 1:(\$3/100) with lines plot "data" using (log(\$1)):(log(\$2)) with lines

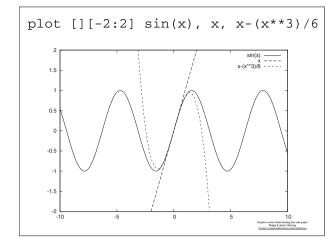
set logscale set logscale x set logscale y unset logscale unset logscale x unset logscale y

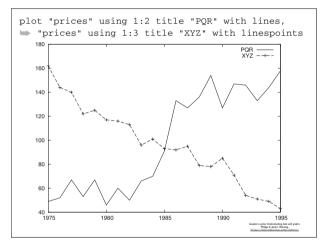
plat in action Understanding data with grapi Philipp K. Janert. Manning.



Plotting Unix /etc/passwd

at:x:25:25:Batch jobs daemon:/var/spool/atjobs:/bin/bash daemon:x:2:2:Daemon:/sbin:/bin/bash ftp:x:40:49:FTP account:/srv/ftp:/bin/bash games:x:12:100:Games account:/var/games:/bin/bash ldap:x:76:70:User for OpenLDAP:/var/lib/ldap:/bin/bash lp:x:4:7:Printing daemon:/var/spool/lpd:/bin/bash mail:x:8:12:Mailer daemon:/var/spool/clientmoueue:/bin/false man:x:13:62:Manual pages viewer:/var/cache/man:/bin/bash mysql:x:60:108:MySQL database admin:/var/lib/mysql:/bin/false news:x:9:13:News system:/etc/news:/bin/bash ntp:x:74:103:NTP daemon:/var/lib/ntp:/bin/false postfix:x:51:51:Postfix Daemon:/var/spool/postfix:/bin/false sshd:x:71:65:SSH daemon:/var/lib/sshd:/bin/false uucp:x:10:14:Unix-to-Unix CoPy system:/etc/uucp:/bin/bash wwwrun:x:30:8:WWW daemon apache:/var/lib/wwwrun:/bin/false Grupht is action Understanding data w Philipp K. Janert. Manning. http://www.manniira.com/

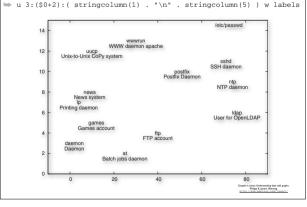


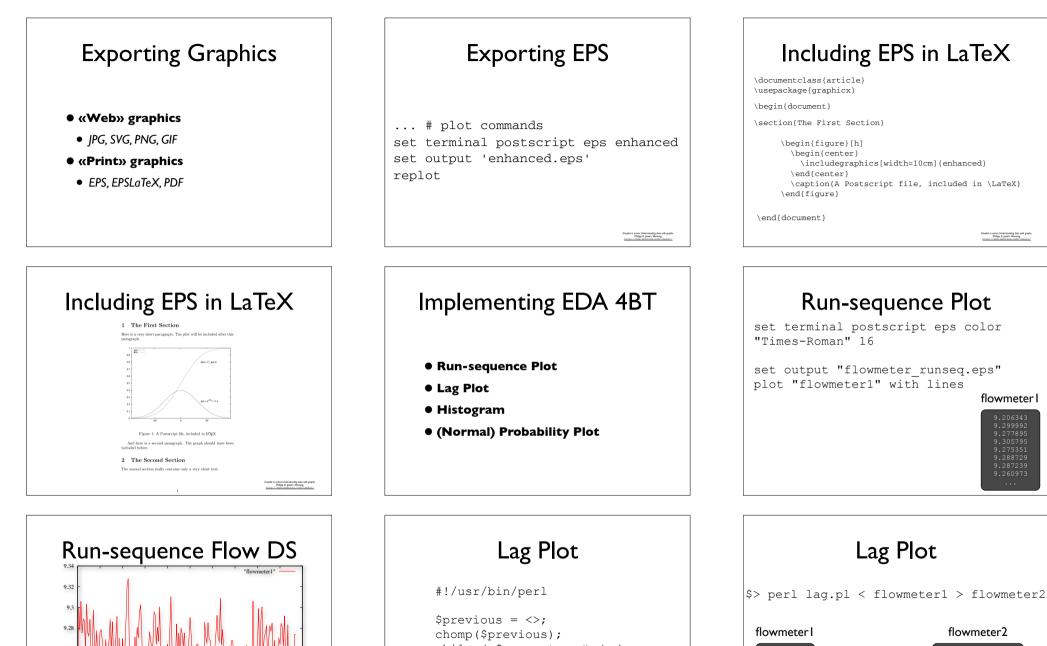


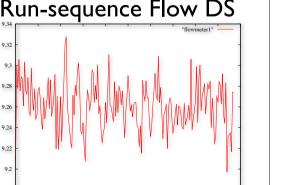
set datafile separator ':'

set datafile commentschar "m"
plot [-20:150][:27] "/etc/passwd"

lot [-20:150][:27] "/etc/passwd"

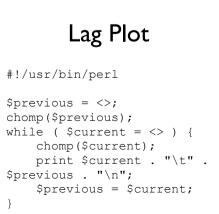


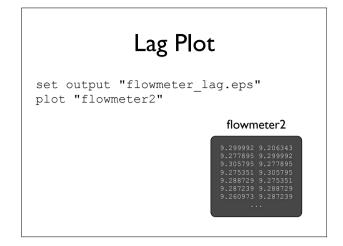


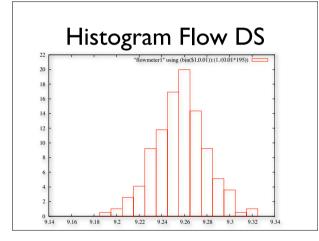


100 120 140 160

9.18



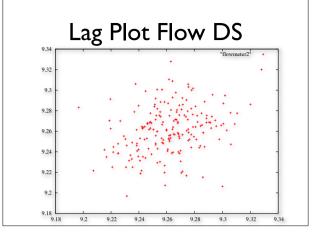




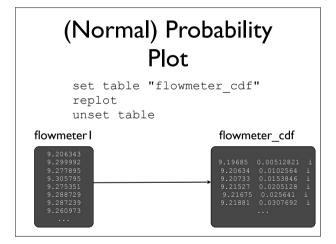
(Normal) Probability Plot

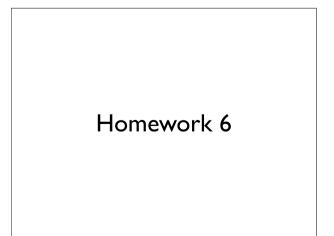
Set output
"flowmeter_isnormal.eps"
plot "flowmeter_cdf" using
(invnorm(\$2)):1 with lines
flowmeter cdf

9.19685 0.00512821 9.20634 0.0102564 9.20733 0.0153846 9.21527 0.0205128 9.21675 0.025641 9.21881 0.0307692 ...



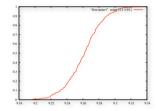
Histogram set output "flowmeter_histogram.eps" bin(x,s) = s*int(x/s) set boxwidth 0.01 plot "flowmeter1" using (bin(\$1,0.01)):(1./(0.01*195)) smooth frequency with boxes 9.206343 9.206343 9.206343 9.206343 9.207395 9.277351 9.208729 9.277351 9.208729

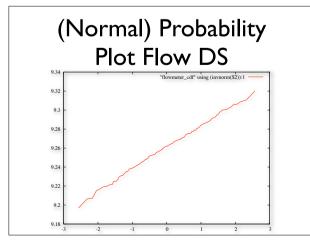




(Normal) Probability Plot

set output "flowmeter_cumulative.eps"
plot "flowmeter1" using 1:(1./195.)
smooth cumulative





Deadline: Tuesday 6 Nov., 17:00

- Step I: Say something *meaningful* about the data located at the METHO class web site:
- Format:
- node x, node y, begin contact, end contact
- One page limit, One graph limit

Deadline: Tuesday 6 Nov., 17:00

- Step 2: download some experimental data:
- <u>http://sensorscope.epfl.ch/index.php/Environmental_Data</u>
- <u>http://fta.inria.fr/apache2-default/pmwiki/index.php?</u> <u>n=Main.DataSets</u>
- Can you *confirm/invalidate* associated publications ? Can you extract *new insight* on the data ?

No page limit, no graph limit, include source code

Step I + Step 2 = One PDF